

Mechanism of Secondary Currents in Open Channel Flows

Shu-Qing Yang

Abstract: This paper describes the conditions for initiation and maintenance of secondary currents in open channel flows. By analyzing the Reynolds equation in the wall-normal and wall-tangent directions, this study reveals that, like other type vortices, the secondary currents are originated from a near-boundary region, its magnitude is proportional to the lateral gradient of near-wall region velocity, and its direction always is always parallel to the solid boundary and points to the region with high velocity. Subsequently, the near boundary flow creeps into the main flow and drives circulation within a region enclosed by lines of zero total shear stress, then leads to anisotropy of turbulence in the main flow region. Typical secondary currents in open channel flows are discussed, such as that induced by sidewall, sand ridges/sediment stripes, compound channel flows etc, all cases support the new mechanism.

Keywords: Secondary flow, Channel flow, Reynolds stress, Shear flow, Three-dimensional flow.

Introduction:

River channels often possess longitudinal bed forms that are aligned parallel to the mean flow direction (Karcz 1973), these longitudinal bed forms take the form of periodic, lateral variations in bed texture (termed sediment strips) and bed topography (termed sand ridges). These phenomena have been widely observed in geophysical flows, for example in gravel-bed rivers (Tsujiimoto 1989, Sambrook Smith and Ferguson 1996), ephemeral streambeds (Culbertson 1967), continental shelves (Kenyon 1970 and Karl 1980). The researchers found from natural flows that the existence of ridge is always associated with secondary currents (Colombini 1993). Laboratory observations (Nezu et al. 1988) confirmed that the formation of strips and ridges is a direct result of cellular secondary currents.

Steady secondary currents in fluid flow have been formally separated into two categories by Prandtl (1952). Secondary flows of the first kind, derived from mean flow skewing or driven by the curvature effect, are in general qualitatively well understood; whilst those of the second kind in straight and non-circular channel, or corner-induced secondary currents are as yet not fully explained. The secondary currents caused by a corner are characterized by flow moving into the apex of corner with a return flow moving outward along the channel boundaries [e.g., Perkins (1970), Gessner (1973), Galletti and Bttaro (2004)]. The corner vortices could damp rapidly from the sidewalls, and therefore the central region in a wide channel remains free from secondary currents (Nezu and Rodi 1985). At its early stage, the stable cellular secondary motion was assumed as the corner-induced vortices, but Nezu, Nakagawa and Tominaga (1985) observed that secondary currents can also be generated without “corners”, if the channel bed is slightly perturbed, or lateral variations in bed topography and roughness can also lead to the formation of secondary currents that, in the central region of a wide channel, are independent of the sidewall effect (Nezu and Nakagawa 1993, Wang and Cheng 2005).

Probably, Einstein and Li (1958) were the first who ascribed the origin of corner-related secondary currents in straight channel flows to the imbalance of the normal Reynolds stresses in

a cross-sectional plane perpendicular to the streamwise direction. Brundett & Baines (1964) and others (see Gerard 1978 for an extensive review) have studied the mechanics of secondary flow using the similar way. Perkins (1970) used Fig. 1 to explain Einstein's postulation about the formation of secondary currents, i.e., how the imbalance of normal Reynolds stress generates the secondary currents are the result of, in which the secondary currents are formed in the main flow region, not the boundary region and the driving force of secondary currents is the variation of turbulent velocity, not the mean velocity. The mechanism of secondary currents by Einstein and Li has been found invalid to explain the observed experiments by many researchers.

Gessner (1973) questioned the mechanism proposed by Einstein by analyzing his experimental data that shows the anisotropy of the turbulent normal stress did not play a major role in the generation of the secondary currents, because in the streamwise vorticity equation, the effects of the normal stresses highlighted by Einstein are of secondary importance compared with the lateral gradient of Reynolds shear stress. The results by Gessner were surprising since they played down the role of the anisotropy of the turbulent normal stresses assumed by Einstein, but in this assumption turbulence is also the prerequisite condition for formation of secondary currents.

Different from Gessner's postulation, Tamburrino and Gulliver (1999) observed the secondary currents in a "still" water body where the channel water was still, but a moving bed was generated by a belt. Based on the observed secondary currents in still water, they concluded that none of the existing theorem can explain their observation. Bradshaw (1987) reviewed the research in secondary currents and concluded that "we lack basic physical understanding of the effect of mean-flow three dimensionality on turbulence structure". Gavrilakis (1992, p.129) commented "a theory on the flow structures that give rise to the observed mean flow is not yet available". Nezu and Nakagawa (1993) also remarked that "the mechanisms for initiation and maintenance of cellular secondary currents in wide channel flows are not yet well understood...".

Not only the driving force of secondary currents, but the role of boundary for its formation is also under debate. Rodriguez and Garcia (2008) observed the multi-cellular structures in a very wide channel with smooth sidewalls and a rough bed, and they ascribed the multi-cellular secondary currents to the large gradient in roughness between smooth glass wall and the gravel bed. This is consistent with Coor and Tait's (2008) experiment, in which there existed high-speed longitudinal streaks over water-worked gravel beds, they conceived that the origin of secondary currents is the result of variation of bed roughness or topography. Albayrak's (2008) study also implies that the secondary currents may depend on the properties of bed roughness. Obviously, these experimental results are contrary to Einstein's postulation in which there is no direct relation between formation of secondary currents and the variation of bed roughness or topography. Recently researchers shed new light on the role of boundary.

The above brief literature review clearly demonstrates that although recent advances should be acknowledged in understanding the secondary flow mechanics and their interrelations with other river processes, there are still many knowledge gaps that have to be addressed (Nikora and Roy, 2010). The main objectives of this study are to: (1) investigate the driving force for the secondary currents; (2) clarify whether the boundary plays a crucial role for the formation of secondary currents; (3) understand the interactions of secondary currents, sediment transport and morpho-dynamics.

Theoretical considerations

For steady and uniform flows, the continuity equation is

$$\frac{\partial V}{\partial y} + \frac{\partial W}{\partial z} = 0 \quad (1)$$

the momentum equation in x-direction can be written as follows (Yang, 2007)

$$\frac{\partial(\rho UV - \tau_{xy})}{\partial y} + \frac{\partial(\rho UW - \tau_{xz})}{\partial z} = \rho g S \quad (2)$$

where U , V and W represent time-average velocities in x , y and z directions, respectively (see Fig. 2), S is the energy slope, g is the gravitational acceleration. u , v and w are turbulent velocity fluctuations. $\tau_{xy} = \mu \partial U / \partial y - \overline{\rho uv}$ and $\tau_{xz} = \mu \partial U / \partial z - \overline{\rho uw}$ in which μ = dynamic viscosity, ρ = fluid density, the over-bar denotes time-averaged values.

To estimate the influence of boundary, for a uniform flow with an arbitrary bed as shown in Fig. 2, the above equations can be alternatively rewritten in the following way:

$$\frac{\partial V_n}{\partial n} + \frac{\partial W_t}{\partial t} = 0 \quad (3)$$

$$\frac{\partial(\rho UV_n - \tau_{xn})}{\partial n} + \frac{\partial(\rho UW_t - \tau_{xt})}{\partial t} = \rho g S \quad (4)$$

where n and t denote the wall-normal and wall-tangent directions. Likewise, the flow region shown in Fig. 2 can be divided into many parts by stripes normal to the boundary. For some convex parts, the divided regions are marked with “I”; for the concave parts the divided regions are marked with “II”; and the remaining parts are marked with “III”.

For every stripe, both Eqs. 3 and 4 are valid. By noticing that the gradient in the wall-tangent direction is much less than the gradient in wall-normal direction, one can simplify Eq. 4 as follows:

$$\frac{\partial(\rho UV_n - \tau_{xn})}{\partial n} = \rho g S \quad (5)$$

$$\frac{\partial(\rho UW_t - \tau_{xt})}{\partial t} \approx 0 \quad (6)$$

Eqs. 5 and 6 convert the partial differential equations into ordinary differential equations for each sub-region enclosed by the boundary normal lines. In order to integrate Eq. 5 along a stripe, one needs to determine its upper limit that is obvious for sub-region “I”, but obscure for sub-region “II”. For the latter case, Yang and his coworkers (Yang and McCorquadale 2004, Lim and Yang 2005, Yang and Lim 2005, Yang, Lim, and McCorquadale 2005) suggested that most of turbulent energy contained in any control volume will be converted to the heat on its nearest boundary, which has the shortest relative distance from the control volume to the boundary, and the relative distance is the ratio of geometrical normal distance (L) to the roughness height (d_{50}) or thickness of viscous sublayer (ν/u_*). Based on this, Fig. 3 is used to demonstrate the determination of upper limit of Eq. 5, in which the flow region is divided into many parts by dashed lines, and the dotted line in the corner can be obtained by equating the relative distances to the side wall and bed (Yang and Lim 1997, Yang and McCorquadale, 2004):

$$\frac{L_b}{\nu / u_{*b}} = \frac{L_w}{\nu / u_{*w}} \quad (7)$$

where L = geometrical distance from the dotted line (or division line) to the boundary; ν = kinematical viscosity; u_* = shear velocity and the subscripts b and w refer to the bed and wall, respectively. Above the division line, the turbulent energy will be dissipated on the wall, and

below it on the bed by friction. The slope of the division line $k = L_w/L_b$ can be theoretically determined by (Yang and Lim, 1997):

$$k^3 + (h/b)k - 2 = 0 \quad (8)$$

where h = water depth, and b = channel width. For a very wide channel, $h/b \approx 0$, k approaches $\sqrt[3]{2} = 1.26$. This means that a pair of secondary currents exist on both sides of $z = 1.26y$.

Tracy (1965) measured the secondary currents in a channel with $h/b=0.156$, his results confirm that a pair of secondary cells exist in the corner and two of them can be roughly separated by a straight line $z = 1.2y$. Tracy's (1965) measured Reynolds shear stress profiles show that if $z > 1.2h$, Reynolds shear stress is independent of z and follows the standard linear line

$$-\frac{\overline{uv}}{u_*^2} = 1 - \frac{y}{h} \quad (9)$$

but if $z < 1.2h$, the Reynolds shear stress no longer follows Eq. 9.

For a very wide channel, Knight and Patel (1985) and Rhodes and Knight (1994) found that the measured mean sidewall shear stress (i.e., $\bar{\tau}_w / \rho ghS$) is constant and equal to 0.63 (see Fig. 4), the mean sidewall shear stress is defined by

$$\bar{\tau}_w = \rho g S \frac{A_w}{p_w} \quad (10)$$

where p is the wetted perimeter, A is the area of water, subscript w denotes the sidewall. As $p_w = h$, then A_w can be determined from Eq. 6 as follows

$$A_w = 0.63h^2 = \frac{1}{2}(1.26h)h \quad (11)$$

as area of a triangle is half of the base multiplied by the height, the division by 2 in Eq. 7 implies that the area affected by sidewall is triangular, and the length of right triangle is $1.26h$ that is precisely the value observed by Tracy. Hence, it can be inferred from the datasets by Knight and his co-workers that the sidewall only affects a triangular flow region. Different experimenters observed the existence of critical value of 1.26, which elucidates that the spread of corner-related secondary current cell is within $z < 1.26h$. Thus, the upper limit of Eq. 5 determined is in accordance with the experimental data, and flow region shown in Fig. 2 can be divided into parts I, II and III, dependent on the boundary curvature.

Secondary currents in near boundary region

Yang and Lim (1997) have investigated the velocity, Reynolds shear stress and turbulent velocity intensity profiles along boundary wall normal lines. In this study, the momentum equation in the direction tangent to the boundary surface will be examined. Eq. 6 becomes zero in the layer very close to the boundary, by integrating Eq. 6 with respect to z , one obtains

$$UW_t - \nu \frac{\partial U}{\partial t} + \overline{uw_t} = 0 \quad (12)$$

In a laminar flow where $\overline{uw_t} = 0$, Eq. 12 reads $\nu \partial U / \partial z - UW_t = 0$ or

$$W_t = (\nu \partial U / \partial z) / U \quad (13)$$

Eq. 13 shows that a velocity W_t can be generated once $\partial U / \partial z \neq 0$ or there exists the lateral velocity gradient. This inference indicates that the secondary currents exist in laminar flows if there is a lateral velocity gradient, and this contradicts the conventional theory that secondary flow is generated by anisotropic turbulence. One of the evidences could be the secondary currents observed by Tamburrino and Gulliver (1999), in their experiment, although the net velocity is zero, there exist the near bed velocity induced by the moving belt. It is understandable

that the velocity at the solid sidewalls must be zero, thus $\partial U/\partial z \neq 0$, and W_t is generated and this residual flow will form the secondary currents and eventually affect the flow stability, this is why the secondary currents can be formed even in still water without net flows, or the secondary currents is caused by the variation of near bed velocity.

For turbulent flows, the viscous effect is negligible, thus in the boundary region, Eq. 12 can be simplified as follows:

$$-UW_t - \overline{uw_t} = 0 \quad (14)$$

By drawing an analogy with the coefficient of molecular viscosity in Stokes's law, Boussinesq (1877) correlated the Reynolds shear stress and strain rate as follows:

$$-\overline{uw_t} = \varepsilon_t \frac{\partial U}{\partial z} \quad (15)$$

where ε_t is the transverse turbulent eddy viscosity. Eq. 15 demonstrates that the Reynolds shear stress $-\overline{uw_t}$ exists if $\partial U/\partial z \neq 0$. Inserting Eq. 15 and 14 gives

$$W_t = \frac{\varepsilon_t}{U} \frac{\partial U}{\partial z} \quad (16)$$

Eq. 16 states that the component of secondary currents W_t is driven by the velocity gradient, i.e., $\partial U/\partial z$, and the largest W_t always exists in the boundary region where U is minimum and $\partial U/\partial z$ maximum, this implies that the secondary currents come from the boundary region.

It is well known that in the viscous sublayer, the velocity profile can be approximated by $U/u_* = u_*y/\nu$, thus the near bed velocity U is proportional to the shear velocity $u_{*(z)}^2 (= \tau/\rho)$, then Eq. 16 reads that the non-uniform distribution of boundary shear stress [$\partial \tau/\partial z \neq 0$] leads to the non-zero W_t , or

$$W_t = \frac{\varepsilon_t}{\tau_{(z)}} \frac{d\tau_{(z)}}{dz} = \varepsilon_t \frac{d \ln \tau_{(z)}}{dz} \quad (17)$$

Therefore, one can say that if there exists a lateral variation of near-wall velocity/boundary shear stress in a channel, secondary flows will be induced and non-uniform distribution of boundary shear stress can be observed. Many researchers, such as Knight and Patel (1985) and Rhodes and Knight (1994), observed the existence of secondary flows in a wide channel and also the periodic variation of boundary shear stress. It can be inferred from Eq. 17 that in such case, there exists the non-zero lateral velocity gradient, i.e., $\partial U/\partial z$ that triggers the secondary currents and lateral variation of boundary shear stress.

Eq. 16 or 17 indicates that positive (rightward) W_t will be formed if velocity U or τ increases with z , vice versa. In other words, the direction of near-bed secondary flow always moves from low velocity point to high velocity point, this is why corner-related secondary flow always moves outward along the channel boundaries (see Fig. 3) because the flow at the apex is stagnant, i.e., $U = 0$.

For a homogenous channel boundary, the velocity gradient $\partial U/\partial z$ in the region $z > 1.26h$ is much smaller relative to the corner region, this is why experimenters cannot detect the secondary currents in the central region (Nezu and Nakagawa, 1993, p126). But, secondary currents are discernible at the same region when non-zero $\partial U/\partial z$ is induced by some disturbances on the bed (artificial ridges) or non-uniform distribution of roughness, this experimental result confirms that the wall-tangent velocity W_t is the result of $\partial U/\partial z$.

Eq. 17 shows that the near-wall secondary flow always moves towards the place where the shear stress is relatively higher (i.e., $\partial \tau/\partial z > 0$), and the maximum or minimum boundary shear stress always occurs at the point where $W_t = 0$, or the upward/downward secondary flow

occurs. Eq. 17 also reads that if the boundary shear stress remains constant, then the secondary flow disappears, or if an experimenter cannot detect the existence of secondary flow in a flow region, he can conclude that the boundary shear stress is constant laterally.

It is interesting to know the location of maximum and minimum boundary shear stress, this can be determined from Eq. 17 by differentiating both sides with respect to z , i.e.,

$$\varepsilon_t \frac{d^2 \tau}{dz^2} = \tau \frac{dW_t}{dz} + W_t \frac{d\tau}{dz} \quad (18)$$

Eq. 18 shows that $d^2\tau/dz^2$ is positive when the secondary flow is upward, negative when downward. Mathematically, this means that the relative maximum boundary shear stress always occurs at the place over which the flow is downward; the relative minimum boundary shear stress appears at the place where the secondary flow is upward.

Webel and Schatzmann (1984) measured the lateral eddy viscosity ε_t in a smooth rectangular channel, and they found that $\varepsilon_t / (u_* h)$ is constant and equal to 0.177, thus Eq. 17 can be rewritten as follows:

$$\frac{W_t}{u_*} = \frac{\varepsilon_t}{u_* h} \frac{h}{\tau} \frac{d\tau}{dz} = 0.177 \frac{h}{\tau_b} \frac{d\tau_b}{dz} \quad (19)$$

Yang and Lim (1997, 1998) shows that the boundary shear stress along a corner can be simplified as a linear distribution, i.e., $d\tau/dz \approx \rho g S$ and $\tau < \rho g h S$ where ρ is density of fluid, h = water depth in Fig. 3, thus

$$\frac{W_t}{u_*} > 0.177 \quad (20)$$

In experimental conditions, $U/u_* = 15 \sim 20$, where U is the cross-sectional average velocity, if we assume $U/u_* \approx 17.7$, then Eq. 20 gives the magnitude of secondary flow as

$$\frac{w}{U} \approx 1\% \quad (21)$$

The theoretical result shown in Eq. 21 is consistent with experimenters' observations (Nezu and Nakagawa, 1993).

The above theoretical results show that the secondary currents origin from the boundary region, this is consistent with the reported facts, for example wrecks on the bed indeed yield the secondary currents and ridges which had been observed and reported. Karcz (1973), Schlichting (1979) and McLean (1981) confirmed that a small isolated obstacle on a river bed, such a rock, a wreck or a boulder, can trigger secondary currents that create a sand ribbon in the vicinity, indicating that the origin of cellular secondary currents and sand ribbons in river flows comes from the near-bed region, rather than the main flow region. These observed phenomena confirm that secondary currents are initiated from the boundary region, rather than the main flow region.

As the significant difference between this study and Einstein and Li's (1958) assumption is whether the secondary currents are originated from the main flow region or boundary region, it is useful to justify the two methods using experimental data. In Fig. 3, if the channel is very wide, then no secondary currents exist in the central region. Now, let's imagine that if the boundary is changed slightly by lowering/rising "AB" or a triangular roughness at "C", thus $\partial U / \partial z \neq 0$ at these points. From Eqs. 16 and 17, we know immediately that the secondary currents will be generated, but Einstein and Li's (1958) theorem cannot certain its generation especially when the roughness size is too small to affect the main flow turbulence. Nezu and Nakagawa (1993) conducted a measurement in a channel with aspect ratio $b/h = 10$, they measured a smooth

channel first and the results are shown in Fig. 5a, and then they put a very small triangular longitudinal ridge at the position of the minimum bed shear stress over the smooth bed, and immediately a secondary flow was triggered (see Fig. 5b). Eq. 16 or 17 shows that the magnitude of secondary flow only depends on the velocity gradient, the higher velocity gradient, the stronger secondary flow can be triggered. Hence, it is understandable that larger and isolated bed roughness elements like sand ridges can intensify the strength of secondary flow. This is consistent with the experimental observation shown in Figs. 5a and 5b. But it should be stressed that, from Eq. 16, the magnitude of secondary flow does not increase with the size of roughness, but depends on its distribution. Secondary currents in Fig. 5b may disappear if the roughness elements are uniformly distributed along the wetted perimeter. This is direct evidence showing that the source of secondary currents comes from the near bed region.

Secondary currents in main flow region

For every stripe in Figs. 1 and 3, Eqs. 5 and 6 are strictly satisfactory in the near boundary region, but they become approximate in the region away from the boundary, but similar conclusion can be drawn that the flow parallel to the boundary or W_t can be generated if there exists a lateral variation of streamwise velocity U . It would be interesting to investigate how the W_t changes its moving direction and forms secondary flow cells in the main flow region.

As fluid particles cannot penetrate, symmetrical lines and free surface, thus the wall-tangent flow W_t has to change its direction when it encounters these interfaces, because in Fig. 3, or the total shear stress on the upper limit of each stripe must be zero, or the division line functions like the free surface or symmetrical lines, thus along the dotted lines in Fig. 3, one has

$$UV_n - \nu \frac{\partial U}{\partial n} + \overline{uv_n} = 0 \quad (22)$$

Eq. 22 expresses the total shear stress at the upper limit of each stripe, it shows that at the division line in Fig. 3 the Reynolds shear stress may not be zero if V_n is non-zero. As the division functions like a free surface, thus the wall-tangent flow W_t must turn its direction when the secondary flow meets the solid wall, free surface or the division line, eventually the secondary flow is confined in the region delimited by these lines, and W_t has to be circulated or the vortex is formed. Hence, the near-bed velocity W_t drives the corner-related secondary currents in Fig. 3.

As the near boundary W_t drives a circulation, the wall-normal velocity V_n will be induced in the main flow region, and its influence on the Reynolds shear stress can be found from the integration of Eq. 5, i.e.

$$-\frac{\overline{uv}}{u_*^2} = \left(1 - \frac{y}{h}\right) + \frac{UV_n}{u_*^2} \quad (23)$$

Eq. 23 states that $-\overline{uv}/u_*^2$ varies with the secondary currents, likewise, the Reynolds normal stresses u , v and w will change due to the secondary currents, or the imbalance of Reynolds normal stress appears, thus the anisotropy of turbulence in main flow region is the result of secondary currents, not the cause of secondary currents. Eq. 23 indicates that only V_n , not W_t plays roles for the momentum transfer that will affect mass and energy transfer, this is consistent with Nezu and Nakagawa's (1993) observations shown in Table 1.

However, previously the role of imbalance of normal Reynolds stress in the main flow region has been overestimated, but the role of boundary has been severely underestimated for the formation of secondary currents. Obviously, if the boundary region is the source of the secondary currents, then the secondary flow will be the maximum in the wall region, and the secondary flow in the main flow region will be smaller in magnitude as it is induced by the flow

in the near-wall region. Likewise, if the main flow region is the source of secondary currents as shown in Fig. 1, then the magnitude of secondary flow in the main flow region must be higher than that in the near wall region. Therefore, one can determine the source of secondary currents by identifying location of its maximum velocity of secondary currents. By examining the measured secondary currents in the literature, one can conclude the maximum velocity always appears in the near wall region, and more precisely the maximum velocity of secondary flow always corresponds to the place where $\partial U/\partial z = \max$.

As the secondary currents in a very wide channel can be also generated by sand ridges, the theorem proposed by Einstein and Li (1958) cannot obviously explain the discovery, thus a new explanation is needed. To explain the secondary currents induced by non-corner effects, Nezu and Nakagawa (1984) postulated that the mild peak in the lateral bed shear stress caused by the presence of the corner secondary flow produces a spanwise variation of the bed-load transport that leads to the formation of a sand ridge. Probably, this is the first time that attention was shifted from the main flow to the boundary region, and from the imbalance of turbulence to the boundary shear stress.

In Fig. 3, if the boundary at “AB” is lowered, then the rectangular channel becomes a compound channel. Most natural rivers have floodplains that extend laterally away from the main river channel at a gentle gradient or in a series of terraces. There is always some discharge in the main channel all of the time, but the floodplains are dry most of the time except during times of flood when they perform a vital function. A number of experimental and numerical studies have been performed and a comprehensive review can be found in Shiono and Knight’s (1991) work. All these research works show that secondary currents occur in compound channels, and the strength is intensified at the interface between the main channel and the floodplains. It is useful to explain how secondary currents are generated in such case.

For a very wide floodplain without water depth variation, $\partial U/\partial z \approx 0$, thus the secondary currents in this region are too weak to be discernible, so Eq. 16 can explain the disappearance of secondary currents in such case. In the main channel, i.e., from “A” to “B” the velocity U at both sides of these apexes are different, thus $\partial U/\partial z \neq 0$, according to Eq. 16, a secondary flow will be generated. As the maximum shear stress occurs at “A” and “B”, i.e., the interface between the main channel and floodplain (Yang et al, 2005), Eq. 17 indicates that the near-wall secondary flow will move towards these apexes from both sides. This is why the secondary currents can be observed in compound channels.

Therefore, one can conclude that Eqs. 16 and 17 provide a good explanation for the generation of secondary currents, i.e., once there exists a non-zero lateral velocity gradient, the secondary currents will be formed, and the velocity variation can be caused by lateral variation of isolated roughness (see Fig. 3, or sand ridge induced), solid walls (corner-induced), non-uniform roughness distribution and water depth variation (compound channel), etc. These different types of secondary currents have been widely observed and documented (see Nezu and Nakagawa, 1993). Based on the aforementioned discussion, it can be inferred that the secondary currents would have the following characteristics:

- 1) Secondary currents will be generated if there exists a lateral variation of streamwise velocity in the near bed region, or $\partial U/\partial z \neq 0$.
 - 2) The near wall secondary flow always moves towards a place where the velocity U is higher.
 - 3) The secondary currents will never penetrate division lines, solid boundary and free surface.
- Thus lines with zero shear stress τ delineate the size and shape of secondary currents.

This study ascribes the source of secondary currents to the flow near boundary. In other words, we can say that the secondary currents like other eddies or vortices come from the boundary region, it is well known that the turbulent eddies is produced in the boundary layer by the wall-normal velocity gradient, i.e., $\partial U/\partial y \neq 0$; the large streamwise vortices after spike dikes in river engineering are also induced by the boundary variation, or $\partial U/\partial x \neq 0$.

Quantitative model for secondary currents by sand stripes

As a lateral variation of roughness can generate the non-zero $\partial U/\partial z$, it is understandable that secondary currents will be formed. McLelland et al (1999), Wang and Cheng (2005) have confirmed the existence of secondary currents experimentally, their data show that at a joint of differential bed roughness, the velocity gradient or the lateral secondary velocity reaches its maximum, for example at point 2 of Fig. 6. In other words, when the difference of roughness on both sides of point 2 becomes smaller and smaller, the secondary currents will be weaker and weaker. According to Eq. 16, if the difference vanishes, the secondary flow above point 2 will disappear.

Eqs. 9 and 23 states that the standard linear distribution (SLD) of Reynolds shear stress can be observed when and only when the momentum flux $UV_n = 0$; the measured Reynolds shear stress will be higher than that predicted by SLD when $UV_n > 0$; and vice versa. In other words, the concave/convex profile of measured Reynolds shear stress interprets the existence of downward/upward secondary currents.

As the near bed velocity U is uneven ($\partial U/\partial z \neq 0$) due to the variation of bed configuration, thus the lateral Reynolds shear stress $-\overline{uw}$ must exist (see Eq. 15). To balance this Reynolds shear stress, non-zero UW must be produced (see Eq. 14), this is why the secondary flow can be observed. Eq. 14 indicates that the shear stress $-UW$ and Reynolds shear stress $-\overline{uw}$ have the same magnitude, but with opposite directions.

The Reynolds shear stress is often modeled by

$$-\overline{uv} = u_* \kappa y \left(1 - \frac{y}{h}\right) \frac{\partial U}{\partial y} \quad (24)$$

where κ = karman constant. It is widely accepted that the parabolic eddy viscosity used in Eq. 24 is valid for a shallow-wide channel. It is natural to ask whether this expression can be extended to explain the velocity dip, to do so, Nezu and Nakagawa (1993, p107) experimentally examined the validity of parabolic eddy viscosity, and they concluded that above the velocity dip, “the value of $-\overline{uv}$ is negative. This feature is consistent with the fact that $\partial U/\partial y$ is also negative in this region; consequently, the eddy viscosity model can be applicable even in 3-D flows”.

Substitution of Eq. 24 into Eq. 23 yields

$$\kappa \xi (1 - \xi) \frac{\partial U^+}{\partial \xi} - U^+ V^+ = 1 - \xi \quad (25)$$

where $\xi = y/h$, $U^+ = U/u_*$, $V^+ = V/u_*$. As mentioned above that the parabolic eddy viscosity is valid in 3-D flows, it is apparent that Eq. 25 is valid to express the influence of secondary current on the velocity distribution.

Eq. 25 demonstrates that if $V = 0$, the classical log-law can be obtained; if $V < 0$, $\partial U^+/\partial \xi = 0$ or (maximum velocity) appears below the water surface; if $V > 0$, the velocity gradient is higher than the gradient of log-law, and subsequently the wake-function is needed to correct the velocity increment. In other words, Eq. 13 states that the velocity follows the log-law if and only if the momentum flux $\rho UV = 0$; the dip-phenomenon occurs if $\rho UV < 0$; the wake-function is needed if and only if $\rho UV > 0$. As secondary currents are ubiquitous in river flow, it is why

Bridge and Jarvis (1985) found the observed velocity in rivers often deviates from the prediction by log-law.

Data Analysis and Verification

The above discussion qualitatively explains how the secondary currents are generated and how the strength, size and directions of a cell of secondary currents are determined. Wang and Cheng's (2005) experimental data are used for verification, their experiment was designed to observe how non-uniform roughness on a bed triggers secondary currents, they conducted their measurement in a 0.6 m wide channel where periodic sediment stripes were covered on the bed laterally (see Fig. 6). The measured parameters include velocities, secondary currents as well as the Reynolds shear stress, thus the comprehensive database can be used to clarify the mechanism of secondary currents and to test the above model.

a) **measured secondary currents in near boundary region:** Wang and Cheng clearly observed the secondary currents as shown in Fig. 6. In their experiment, there exists lateral velocity gradient, because the strips with sediment retard the flow and other strips without sediment correspond high-speed zones. As predicted by Eq. 16, secondary currents were generated and the observed results are shown in Fig. 6. Interestingly, Eq. 16 also predicted that if the lateral gradient of mean velocity U disappears, then the secondary currents appear. This means that if Wang and Cheng's channel bed was fully covered by sediment, then the secondary currents will disappear, this inference is consistent with other's experiments where the channel bed was uniformly covered by sediment, e.g. Yassin (1953). Thus one can conclude that the secondary currents are indeed generated from the boundary, not the main flow region as assumed by Einstein.

Most importantly, Eq. 16 also predicts that the near-boundary secondary flow always moves from a place with lower velocity to another place with higher velocity. Wang and Cheng measured the velocity U along 1-1, 2-2 and 3-3 profiles that are reproduced in Fig. 7. Comparing the measured near-bed velocity at 1-1 and 3-3 profiles, one can see that the former is smaller than the latter (see Fig. 7b where the measured data points are linked by solid lines, thus one can see the lateral gradient of mean velocity). Based on Eq. 16, the near-bed secondary flow must move from "1-1" to "3-3" that drives the clockwise secondary flow. Wang and Cheng's (2005) measurement do confirm this prediction as shown in Fig. 6. Therefore, the measured mean velocity in the near-bed region is sufficient to justify the existence of secondary currents and its flow direction. For this justification, it is unnecessary to use the measured turbulent velocity as Einstein advocated.

b) **measured secondary currents in the main flow region:** The above discussion shows that in the main flow region, the wall-normal velocity plays an important role for the mass, momentum transfer. If the wall-normal velocity is zero, Eq. 23 predicts that the Reynolds shear stress will follow a linear distribution, and Eq. 26 says that the velocity follows the log-law. From Fig. 6, one can see that the wall normal velocity for profile "1-1" is downward (negative); for profile "3-3" (or positive), and zero for profile "2-2". Fig. 7a clearly demonstrates that the measured velocity along "2-2" can be represented by a straight line or follow the log-law. Fig. 8a also shows that, as expected, the measured Reynolds shear stress can be roughly represented by a straight line, but the measured Reynolds shear stress along profiles 1-1 and 3-3 are systematically lower or higher than the linear distribution. These deviations are also consistent with Eq. 23 as the term UV/u_*^2 is negative and positive along the profiles of 1-1 and 3-3, respectively. As predicted by Eq. 23, the measured Reynolds shear stresses in 1-1 and 3-3 follow concave and convex distributions as shown in see Fig. 8a where the data points are linked by solid

lines to show the concave and convex distributions. When the data points in Fig. 8a were re-plotted in the form of $(-\overline{uv} - UV)/u_*^2$ against y/h , all data points can be approximately represented by the standard linear line as shown in Fig. 8b, suggesting that the difference between the measured Reynolds shear stress and the straight line in Fig. 8a be caused by UV/u_*^2 .

c) **Quantitative verification of secondary currents:** Based on the new mechanism of secondary currents, the model to express the strength of secondary currents by sediment stripes has been developed and shown in Eqs. 28-41, in which the components of secondary flow in wall-tangent and wall-normal directions are expressed by Eq. 28 and 33, respectively. The wall-normal and wall-tangent velocities V and W can be calculated using Eqs. 28 and 33. Fig. 9 shows the comparison of measured and predicted velocity components of secondary currents, the agreement is acceptable, indicating that the secondary currents originates from the boundary and the lateral gradient of near-bed velocity is the driving force. This can be seen from Fig. 9 in which the highest near-bed velocity, W , occurs at the place where there exists a sharp variation of stripe roughness, i.e., at “2-2”, whereas at the bottom of “1-1” and “3-3”, the $\partial U/\partial z \approx 0$ as no roughness variation at its vicinity, consequently $W \approx 0$. All these measured data suggests that the larger $\partial U/\partial z$ corresponds to larger W , and smaller $\partial U/\partial z$ corresponds to smaller W , this is what Eq. 16 suggests, i.e., W should be proportional to $\partial U/\partial z$.

The dip-phenomenon always associates with the downward velocity V , thus the wall-normal line through the point of velocity dip can serve as a division line; and another wall-normal line through the mid-point of two neighboring points of velocity-dip forms the other division line. Therefore, based on the locations of velocity dip, $\alpha_0 = 0.42$ was obtained and from Eq. 35 $\alpha = 0.21$, the delineation of secondary currents can be identified. The strength of secondary currents can be determined by Eq. 36 based on the information of velocity-dip. It can be seen from Eq. 9 that the semi-empirical model can reasonably capture the main characteristics of secondary currents caused by sediment stripes.

Conclusions

The mechanism for initiation and maintenance of secondary currents of Prandtl's second kind has been investigated. By examine the Reynolds equation in the boundary region, it reveals that the secondary currents originate from the boundary region, just like the generation of turbulent eddies induced by $\partial U/\partial y \neq 0$ and large streamwise vortices induced by $\partial U/\partial x \neq 0$; the lateral variation of streamwise velocity ($\partial U/\partial z \neq 0$) yields the secondary currents. The secondary currents generated by non-uniform distribution of roughness are investigated to test the new theorem quantitatively and qualitatively, and good agreement between the measured and predicted results is achieved. The main findings of the present study can be summarized as follows:

1. Secondary flow W in the near bed region must be generated to balance the transverse Reynolds shear stress $-\rho \overline{uw}$ that is always associated with the lateral variation of streamwise velocity. Consequently the secondary flow causes a circulation in an allowable domain, and turbulence anisotropy appears in the main flow region of the domain. Therefore, the driving force to initiate the secondary currents of Prandtl's second kind is the transverse Reynolds shear stress $-\rho \overline{uw}$ or $\partial U/\partial z$.
2. The source of secondary currents comes from the near-wall region, rather than the main flow region. In other words, any small disturbance on the bed flow region may trigger secondary currents, which in turn, results in the anisotropy of turbulence in the main flow region. The interaction of turbulent anisotropy and the secondary flow can be inferred from Eq. 23

theoretically and Table 1 experimentally. Hence, the boundary is not only the primary source of turbulence, but it is also the source of secondary currents; in the main flow region, only V_n plays important role for the mass, momentum and energy transfer, W_t 's role is negligible.

3. The near-bed secondary flow always departs from the low velocity strip to the high velocity stripe. The maximum secondary flow occurs at the place where the near-bed velocity U changes abruptly (e.g. from floodplain to main channel), secondary currents disappear if the near-bed velocity U remains unchanged laterally.

4. The relationship between the boundary shear stress and secondary currents is established, the theoretical result shows that the near-wall secondary flow always moves from the lower boundary shear stress to the place with higher boundary shear stress. Alternatively, the higher boundary shear stress always associated with downflow, but lower boundary shear stress appears at the region with upflows.

5. This study chooses the secondary currents induced by non-uniform distribution of roughness to test the discovery. Although some approximations have been applied to simply the problem, the obtained results agree with the measured data reasonably well, indicating that without using the anisotropy of turbulence, the secondary currents and associated turbulence characteristics can be also captured.

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Table 1. Summary of turbulent structures of cellular secondary currents in open channels (after Nezu and Nakagawa, 1993).

Type	Upflow region ($v > 0$)	Downflow region ($v < 0$)
Primary mean velocity	Low	High
Bed shear stress	Low	High
Turbulence intensity farther from the bed	High	Low
Reynolds shear stress farther from the bed	High	Low
Suspended load	High	Low
Bed load	Low	High
On water surface	Boil lines, divergence	Foam lines, convergence
River bed form	Ridges	Troughs
Bed roughness	Fine sand, smooth bed strips	Coarse sand, rough bed strips

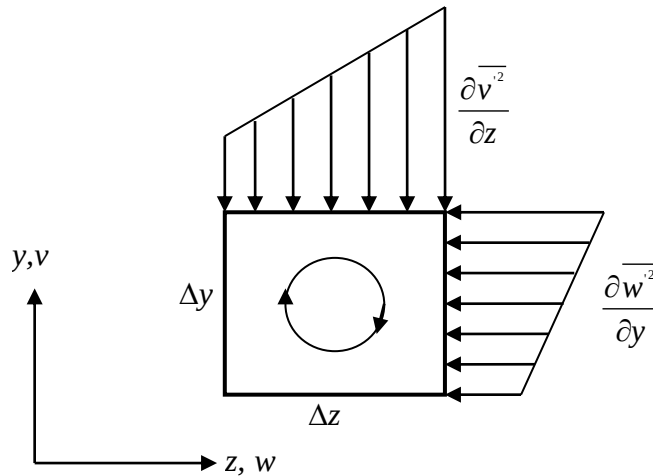


Fig. 1. vorticity production by the imbalance of normal shear stress, after Perkins (1970). This was used to support Einstein's postulation that formation of secondary currents is caused by anisotropic turbulent velocity, and has no direct relationship with the mean flow and boundary.

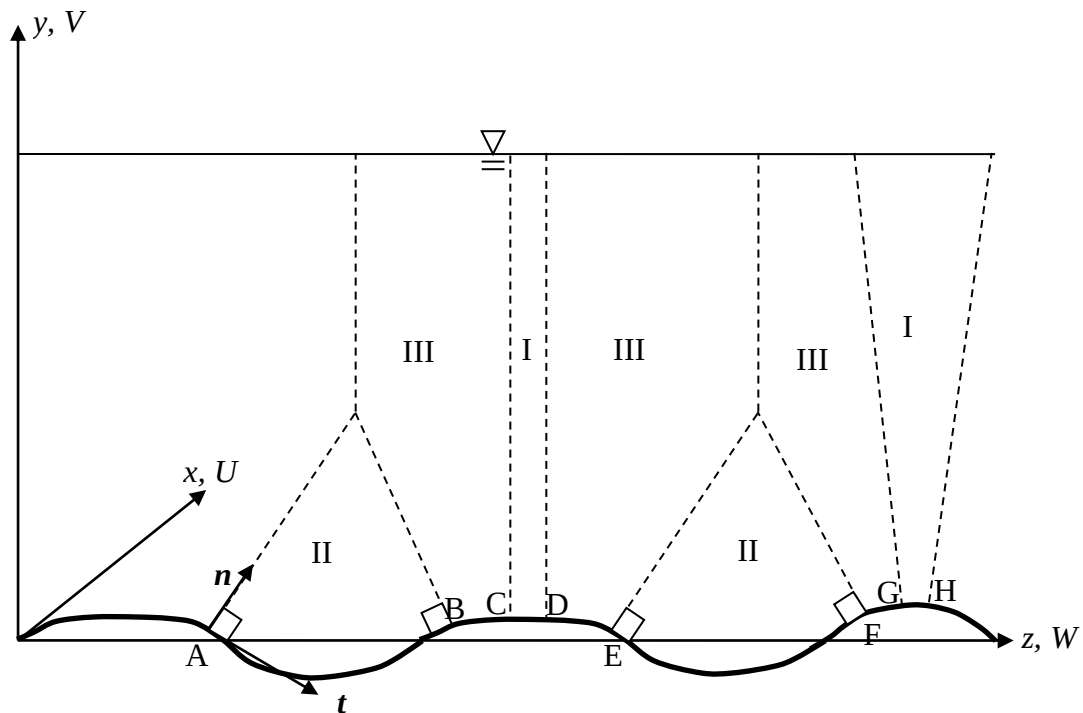


Fig. 2. Sketch of flow configuration, a flow region is dividable by the boundary normal lines and different types of sub-region can be determined, i.e., I, II and III based on whether the normal lines are intersected

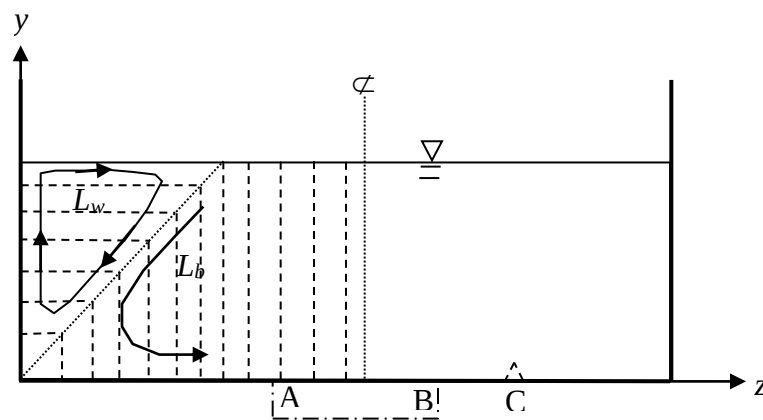


Fig. 3. Flow region is divided by the stripes normal to the boundary, an arbitrary point in the flow corresponds to two normal lines from sidewall or bed, then the turbulent characteristics of turbulence at this point depends on the “nearest boundary” that is the ratio of geometrical normal distance to the sidewall or the bed to boundary roughness or viscous sublayer thickness, the dotted line that has same relative distances to the wall and bed can be used approximately as upper limit of Eq. 5.

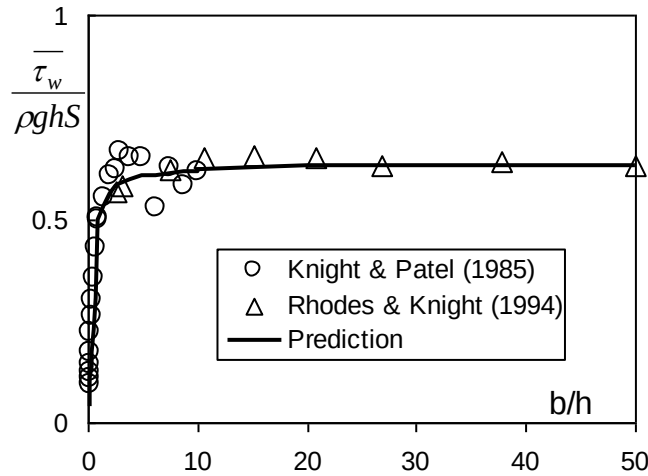


Fig. 4. Comparison of measured and predicted sidewall mean shear stress versus the aspect ratio for smooth open channel and duct flows (Yang and Lim, 1997), it shows that the normalized shear stress approaches to half of 1.26, implying that the division line in Fig. 3 can be expressed by $z = 1.26y$.

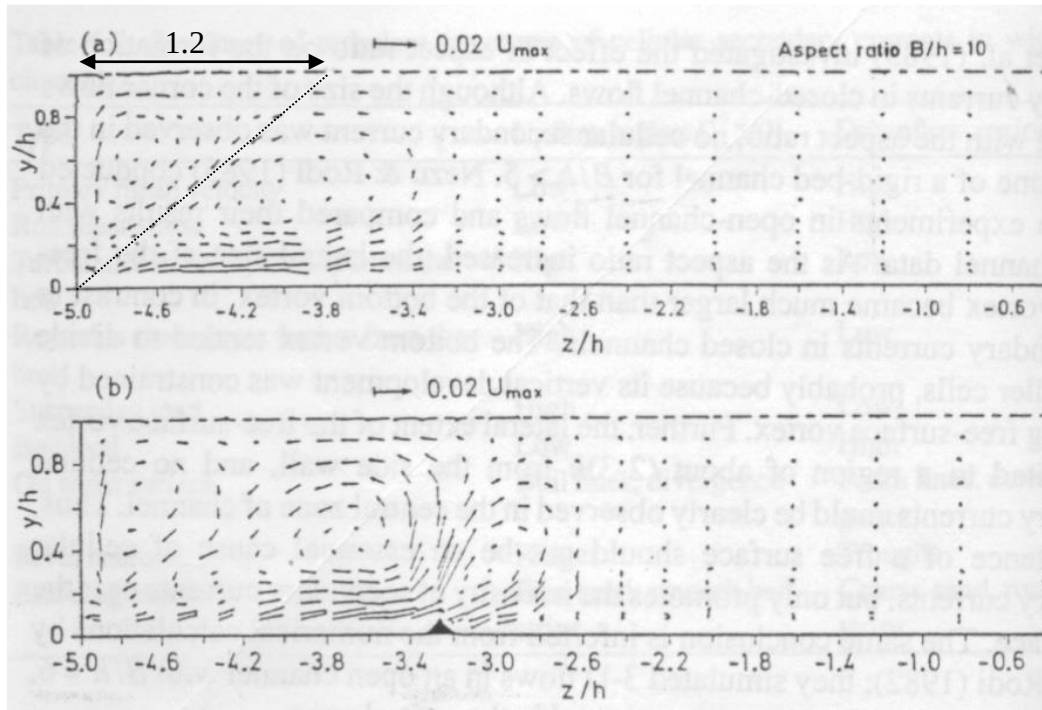


Fig. 5. Velocity vector for cellular secondary currents (a) induced by sidewall where the dotted line is $z = 1.2y$ indicating that the flow is really dividable by the wall-normal lines; (b) introduced by longitudinal ridge, after Nezu and Nakagawa (1993).

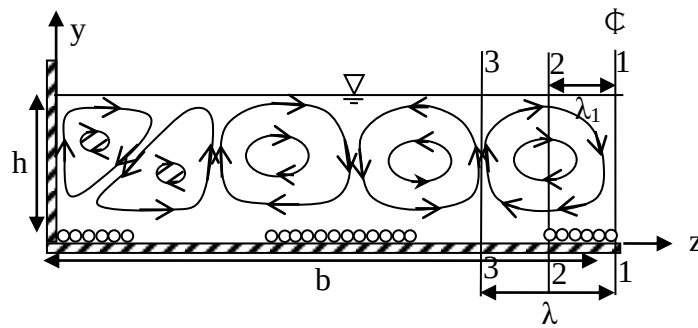


Fig. 6, typical secondary currents induced by sidewall and roughness strips in Wang and Cheng's (2005) experiment, the width of secondary currents is denoted by λ , the width of rough stripe is λ_1 . Along profile 1-1, 2-2 and 3-3, the wall normal velocity is negative (downward), zero and positive (upward)

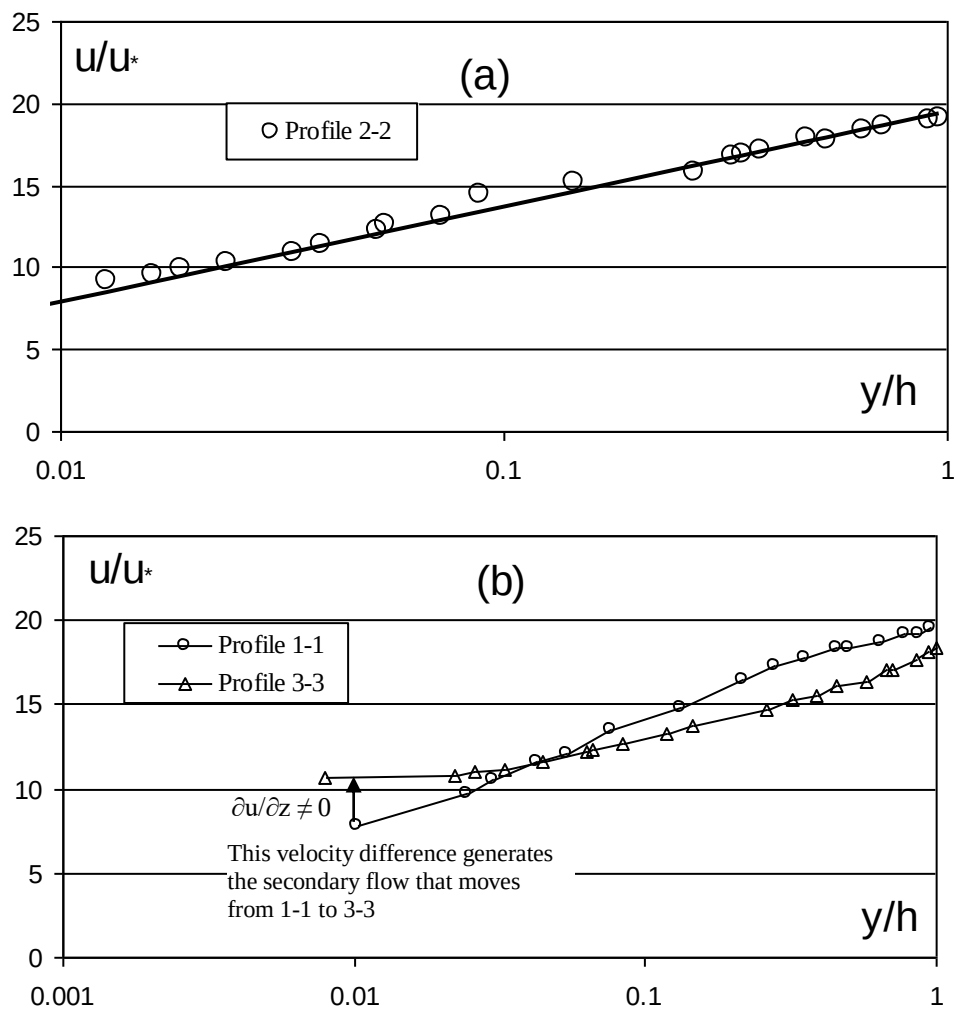


Fig. 7. Measured velocity distributions (a) along profile 2-2 where wall-normal velocity is zero, and the straight line indicates that the velocity follows the logarithmic distribution (b) along profiles 1-1 and 3-3 where the near bed velocity differs from each other, this difference can be seen from the solid lines that links the data points, the difference at the bottom generates wall-tangent velocity W which consequently induces vorticity, data from Wang and Cheng (2005), .

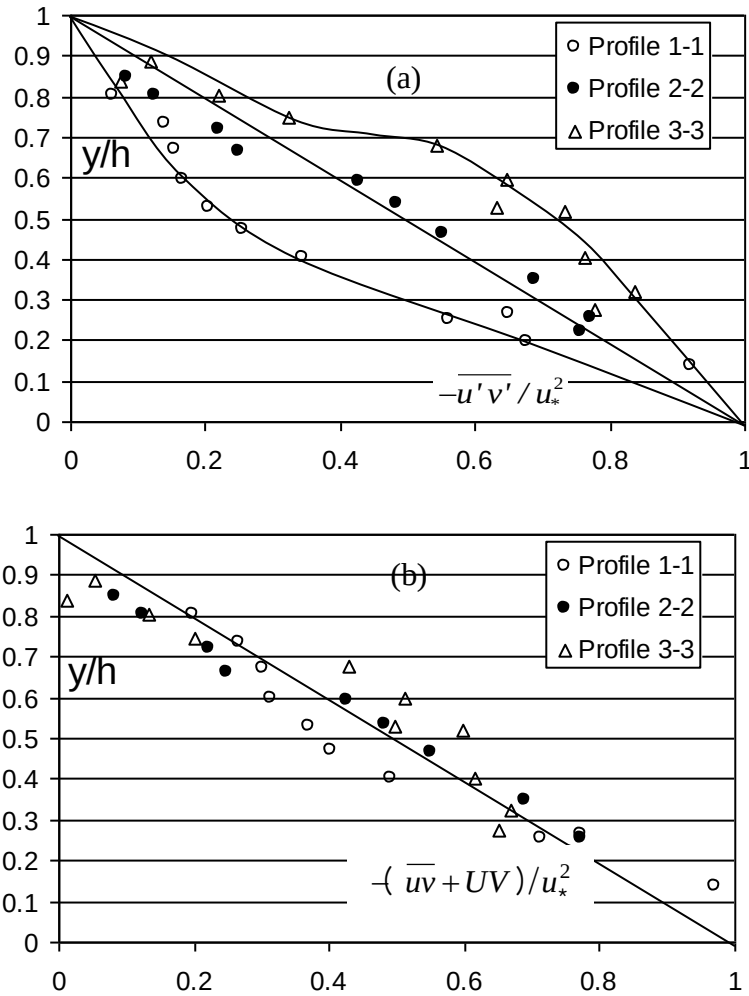


Fig. 8. (a) Measured Reynolds shear stress , the straight line represents the along 2-2 the Reynolds shear stress is linear, the curves shows concave and convex distribution of Reynolds shear stress; (b) measured total shear stress distribution, data from Wang and Cheng (2005), i.e., the total momentum still follow the linear distribution.

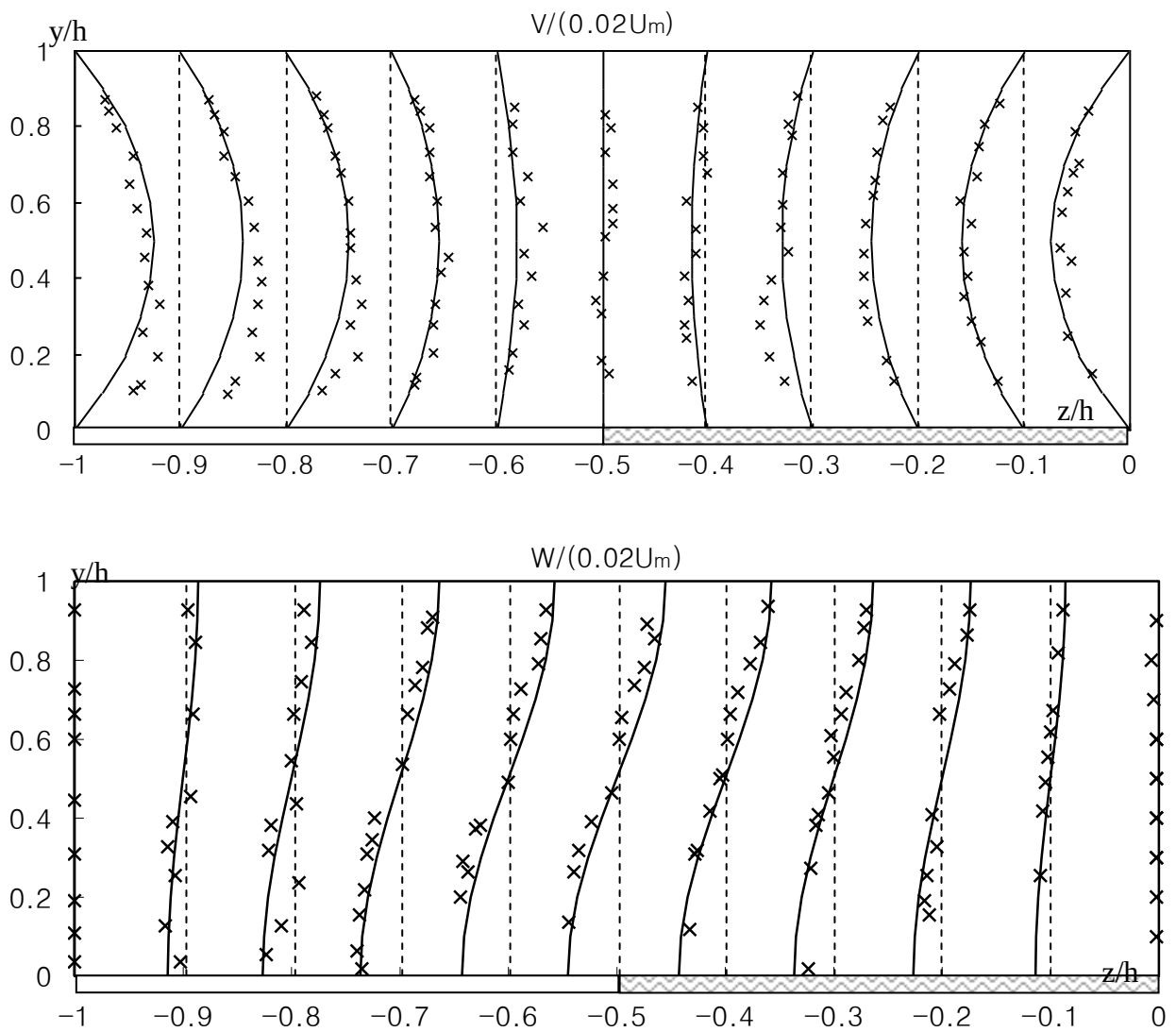


Fig. 9. Comparison between measured and predicted results of V and W , the relative velocity is scaled to the distance between two neighboring dashed lines, which is take to be unity (after Wang and Cheng 2005), the solid lines are calculated from Eqs. 28 and 33.