UNCERTAINTY ANALYSIS OF ROUGHNESS MEASUREMENTS FOR A LARGE-SCALE CANAL

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ABSTRACT

Factors related to the uncertainty of roughness measurements were analyzed in a large-scale canal: the Middle Route Project (MRP) in China. With field observation data of seven canal sections, a formula to estimate the uncertainties of the roughness coefficients was derived, and sensitivity coefficients were determined. Curves illustrating the variations of the sensitivity coefficients and roughness coefficient uncertainties with spaces between water level stations are presented. Exponential fitting formulas were generated. In this study, it was shown that the flow rate measurements were the main source of error and dominated the calculation of the uncertainty. The uncertainty can be reduced by increasing the space between water level stations. This is especially effective within a specific spacing range, which can be identified from the fitting formulas.

Keywords: Roughness; uncertainty analysis; canal; sensitivity coefficient

1 INTRODUCTION

Manning’s roughness coefficient ($n$) is a key design parameter in canal projects. It reflects the amount of frictional resistance water experiences when passing through the canal. The roughness coefficient has a significant impact on the flow conveyance capacity (Kim et al., 2010). For example, in the Middle Route Project (MRP) of the South-to-North Water Diversion Project in China, a 0.001 increase in the roughness coefficient corresponds to a 0.2 m increase in the water level (China Institute of Water Resources and Hydropower Research, 2016). An accurate estimation of the roughness coefficient is essential not only for the verification of the flow conveyance capacity but also for the evaluation of construction and maintenance qualities.

Manning’s roughness coefficient is a derived parameter that can only be acquired indirectly. The value of $n$ is affected by many factors, such as variations in the channel width and depth, sinuosity of the canal, and biological factors. Research has shown that a minor error in field measurements can cause major deviations in the roughness coefficient estimation (Yang et al., 2012). Therefore, in practice, the roughness coefficient is usually estimated via prototype observations. For many large-scale canal projects in China (Chen et al., 2016; Lyu et al., 2009), prototype observations were conducted when the projects were newly constructed. Much work has been done on uncertainty analysis of stream gauging. However, to date, uncertainty estimation has rarely been associated with flow data (Despax et al. 2016), especially related to roughness coefficient estimation (Erhan et al., 2008).

In this study, uncertainty analysis was conducted in the MRP to assess the quality of roughness estimations. Field measurements from 2009, when the project was newly constructed, were analyzed. Error propagation rules of the roughness coefficient measurements were established. The contributions of several main measurement error sources, namely the water levels upstream and downstream and the discharge, were investigated. Suggestions of how to guarantee the proper confidence levels associated with the instrument precision and field measurement scheme are presented.

2 DERIVATION OF THE UNCERTAINTY OF MANNING’S ROUGHNESS COEFFICIENT MEASUREMENT

A straight, uniform canal section is usually chosen for roughness coefficient determination. Two measurement sites are established, one at the upstream end and the other at the downstream end of the canal section. The flow is kept steady during the measurement. The water levels at the two measurement sites and the discharge are measured simultaneously. Manning’s roughness coefficients can be easily calculated using the following equations:

$$Q = \frac{1}{n} \cdot A R^{2/3} S^{1/2} \quad [1]$$
\[ n = \frac{\bar{AR}^{2/3}}{gLc_2} \sqrt{S_0 + \frac{y_2 - y_1}{L} + \frac{1}{A_1^2} - \frac{1}{A_2^2} Q^2} \]  

[2]

in which \( n \) is Manning’s canal roughness coefficient, \( A \) is the flow area, \( R \) is the hydraulic radius, \( Q \) is the discharge, \( S \) is the slope of the hydraulic grade line, \( S_0 \) is the bed slope, \( y \) is the water level, subscripts 1 and 2 denote the sites at the upstream and downstream ends, respectively, \( g \) is the gravitational acceleration, \( L \) is the length between the measurement sites (m), and \( \bar{AR}^{2/3} = (A_{1,2}^{2/3} + A_{1,2}^{2/3}) / 2 \).

It is reasonable to assume that the measurements of \( y_1, y_2, \) and \( Q \) are uncorrelated. It is hypothesized that the uncertainty contribution from \( L \) is negligible compared to those from \( y_1, y_2, \) and \( Q \). Based on the theory of uncertainty propagation for independent variables (Taylor, 1997), the uncertainty of the corresponding Manning roughness coefficient can be estimated as follows:

\[ u(n) = \sqrt{C_1^2 u(Q)^2 + C_2^2 u(y_1)^2 + C_3^2 u(y_2)^2} \]  

[3]

where \( C_1, C_2, \) and \( C_3 \) are the sensitivity coefficients of \( n \) corresponding to \( Q, y_1, \) and \( y_2 \) (Yang et al., 2012), respectively, defined as follows:

\[ C_1 = \frac{\partial n}{\partial Q} = \frac{\bar{AR}^{2/3} c_1}{2gLc_2} \]  

[4]

\[ C_2 = \frac{\partial n}{\partial y_1} = \frac{\partial (\bar{AR}^{2/3})}{\partial y_1} c_2 + \frac{\bar{AR}^{2/3}}{2Qc_2} (1 - \frac{Q^2 B_1}{gA_1^2}) \]  

[5]

\[ C_3 = \frac{\partial n}{\partial y_2} = \frac{\partial (\bar{AR}^{2/3})}{\partial y_2} c_2 + \frac{\bar{AR}^{2/3}}{2Qc_2} (1 - \frac{Q^2 B_2}{gA_2^2}) \]  

[6]

where \( c_1 = \frac{1}{A_1^2} - \frac{1}{A_2^2} \), \( c_2 = \sqrt{S_0 + \frac{y_1 - y_2}{L} + \frac{Q^2}{2gL}} \), \( \var{\partial (\bar{AR}^{2/3})}{\partial y_1} = \frac{B_1 R_{1,2}^{2/3} dR_1}{2 A_1^{3/2} dy_1} \) and \( \var{\partial (\bar{AR}^{2/3})}{\partial y_2} = \frac{B_2 R_{1,2}^{2/3} dR_2}{2 A_2^{3/2} dy_2} \).

Velocity-area methods are commonly used methods to determine the discharge. In these methods, the flow area and velocity are measured and multiplied to obtain the discharge as follows:

\[ Q = vA \]  

[7]

where \( Q \) is the discharge, \( v \) is the velocity, and \( A \) is the cross-section of flow.

Depending on the accuracy required, the width of a canal is divided into a number of vertical segments. In each of these segments, the velocity is measured at one or more points along the depth to get a representative velocity in that segment. If the measurement verticals are placed such that the segment discharges are approximately equal, and if the component uncertainties are equal from vertical to vertical, the relative (percentage) combined standard uncertainty in the measurement may be given by formula [8], which was obtained from the ISO 748 standard (ISO 748, 2007):

\[ u_{rel}(Q) = \left[ u_{rel,m}^2 + u_{rel,b}^2 + u_{rel,d}^2 + u_{rel,slp}^2 + \frac{1}{m} \left( (u_{rel,slh}^2 + u_{rel,slp}^2 + u_{rel,slp}^2 + u_{rel,slp}^2) \right) \right]^{1/2} \]  

[8]

where \( u_{rel}(Q) \) is the relative (percentage) combined standard uncertainty in the discharge, \( u_{rel,m} \) is the uncertainty due to the limited number of verticals, and \( m \) is the number of verticals. \( u_{rel,d} \) is the uncertainty due to the variable responsiveness of the current-meter \( (u_{rel,cm}) \), width measurement instrument \( (u_{rel,bm}) \), and depth sounding instrument \( (u_{rel,sd}) \).\( u_{rel,b} = \sqrt{u_{rel,cm}^2 + u_{rel,bm}^2 + u_{rel,slh}^2} \), in which \( u_{rel,b} \) and \( u_{rel,slh} \) are the relative (percentage) standard uncertainties in the width and depth, respectively; \( u_{rel,slp} \) is the uncertainty in the mean velocity due to the limited number of depths at which velocity measurements are made; \( n \) is the number of depths in the vertical at which the velocity measurements are made; \( u_{rel,c} \) is the uncertainty in the velocity at a particular measuring point in the vertical due to lack of repeatability of the current-meter; and \( u_{rel,sl} \) is the uncertainty in the depth point velocity at a...
particular depth in the vertical due to velocity fluctuations (pulsations) in the flow during the exposure time of the current-meter.

The uncertainty is calculated as follows:

$$ u(Q) = u_{ref}(Q) \times Q $$ \hspace{1cm} [9]$$

Gauge plates can be used to measure the water levels at the upstream and downstream sites. The standard uncertainty estimates $u(y)$ are based largely on the resolution of the instrument.

The expanded uncertainty at the 95% confidence level, $U_{exp}(n)$, is obtained by applying a coverage factor of $k=2$, and thus,

$$ U_{exp}(n) = k \times u(n) $$ \hspace{1cm} [10]$$

The relative combined standard uncertainty is as follows:

$$ U_{exp,rel}(n) = U_{exp}(n) / n \times 100\% $$ \hspace{1cm} [11]$$

3 UNCERTAINTY ANALYSIS OF MRP ROUGHNESS MEASUREMENTS

In 2009, a prototype observation was carried out to measure the roughness of the canal in the Jingshui part of the MPR (Construction and Administration Bureau of South-to-North Water Diversion Middle Route Project, 2009). The test area comprised 7 consecutive sections. The dimensions of the canal sections are shown in Table 1. The discharge was 19.00 m³/s. The lengths of the 7 sections ranged from 568 to 9416 m. The roughness coefficients were derived section-by-section and ranged from 0.0136 to 0.0157.

<table>
<thead>
<tr>
<th>Table 1. Dimensions and roughness coefficients of the canal sections</th>
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<tbody>
<tr>
<td>Section</td>
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Velocity-area methods were used to determine the discharge. The flow area had dimensions of 5.7 m (depth) × 15.5 m (width). A five-point method (using a current-meter on each vertical at 20%, 60%, and 80% of the depth, one as near as possible to the surface, and one at the bed) was used to measure the velocity with a current-meter. Fifteen verticals were used in the gauging (i.e., m=15). The average velocity in the measuring section was 0.2 m/s. The exposure time of each current-meter was 3 min. Current-meters were individually rated. Referring to Annex E of the ISO 748 standard, component uncertainties (percentages) were obtained from the information given, $u_l=4\%$, $u_s=1\%$, $u_b=0.15\%$, $u_k=0.5\%$, $u_r=2.5\%$, $u_w=1\%$, and $u_d=4.0\%$, for all five depth points. Therefore, $u_{rel}=\sqrt{4^2+4^2+4^2+4^2} = 8.94\%$. From formula [8], $u_{rel}(Q)=3.39\%$, and $u(Q)=u_{rel}(Q)\times Q=0.64$ m³/s. Using formula [3], the sensitivity coefficients ($C_1$, $C_2$, and $C_3$) and $u(n)$ for each section are shown in Table 2. The expanded uncertainty at the 95% confidence level, $U_{exp}$, was obtained by applying a coverage factor of $k=2$, and thus, $U_{exp}(n) = k' u_{rel}(Q)=2\times3.39\%=6.78\%$.

Standard metric gauge plates were used to measure water levels at the upstream and downstream sites, which were marked in 5 mm increments and could be read to the nearest millimeter. The resolution of the gauge plate was 0.001 m. Applying a uniform distribution for the resolution error, $u(y)=u(y_0) = 0.001\times1/2/3^{0.5} = 0.00029$ m.

<table>
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<tr>
<th>Table 2. Uncertain analysis results of canal sections</th>
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<tr>
<td>Section</td>
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<tr>
<td>No.</td>
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Table 2 shows that all of the canal sections produced nearly the same levels of uncertainty and relative uncertainty for the roughness coefficient measurements, which were around 0.001 for \( U_{b} \), 0.002 for \( U_{s} \), and 13% for the relative uncertainty.

Although the sensitivity coefficients of the water level \( C_{2} \) and \( C_{0} \) were one order of magnitude larger than that of the discharge \( C_{1} \), the uncertainty contribution of the water level was either slightly or significantly smaller than that of the discharge. Despite this, the discharge measurement was still the main contributor to the uncertainty compared to the upstream and downstream water level measurements, especially for long canal sections.

## 4 APPRAOCHES TO REDUCE THE MEASUREMENT UNCERTAINTY

A relative uncertainty of 12% is not ideal for roughness estimations. Approaches could be found to reduce the measurement uncertainty. From formula [2], the canal section length, water levels at the upstream and downstream measurement sites, and the discharge are the main variables. According to the uncertainty contributions shown in Table 2, the precision of the discharge measurement instrument had a greater influence than that of the water level, especially for long canal sections. Therefore, priority should be given to improving the precision of the discharge measurements.

The main components of the discharge measurement uncertainty can be identified based on formula [5]. In Annex E of the ISO 748 standard, the uncertainty values of various uncertain components are supplied. \( U_{m} \) had a significantly greater weight than the other factors. In the case of the MRP canal sections, other factors, such as \( U_{v} \) (denoting the number of points in the vertical) and \( U_{t} \) (denoting the time of exposure), were already of high quality and were less likely to be improved. Therefore, the combined uncertainty should be deduced by increasing \( m \), i.e., the number of verticals in the velocity-area method. Supposing that \( m \) doubles from 15 to 30, the percentage uncertainties in the measurements of the mean velocity due to the limited number of verticals could be reduced by half, i.e., \( u(m) = 1.5 \)%. From formulas [5] and [6], \( U_{e}(Q) = 2\% \) and \( u(Q) = U_{e}(Q)Q = 0.38 \) m³/s. The uncertainty analysis results using the new \( m \) are shown in Table 3.

### Table 3. Uncertain analysis results of the canal sections with a new value of \( m \)

<table>
<thead>
<tr>
<th>Canal section</th>
<th>Length ( L(m) )</th>
<th>Uncertainty contribution</th>
<th>Uncertainty</th>
<th>Relative uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td></td>
<td>( C_{1}u(Q)^{2} \times 10^{-7} )</td>
<td>( C_{2}u(y_{1})^{2} \times 10^{-7} )</td>
<td>( C_{3}u(y_{2})^{2} \times 10^{-7} )</td>
</tr>
<tr>
<td>1</td>
<td>890</td>
<td>2.440</td>
<td>3.854</td>
<td>3.782</td>
</tr>
<tr>
<td>2</td>
<td>610</td>
<td>2.440</td>
<td>3.316</td>
<td>3.042</td>
</tr>
<tr>
<td>3</td>
<td>568</td>
<td>1.747</td>
<td>1.970</td>
<td>1.921</td>
</tr>
<tr>
<td>4</td>
<td>2215</td>
<td>2.440</td>
<td>0.292</td>
<td>0.270</td>
</tr>
<tr>
<td>5</td>
<td>3124</td>
<td>2.440</td>
<td>0.217</td>
<td>0.199</td>
</tr>
<tr>
<td>6</td>
<td>7631</td>
<td>2.830</td>
<td>0.061</td>
<td>0.052</td>
</tr>
<tr>
<td>7</td>
<td>9416</td>
<td>2.830</td>
<td>0.090</td>
<td>0.079</td>
</tr>
</tbody>
</table>

Curves describing the variation of the uncertainty contributions with the canal section lengths are shown in Figure 1. Curves describing the variation of the relative uncertainty with the canal section lengths are shown in Figure 2. The results for the original and new values of \( m \) are shown in Figures 1 and 2.

Figure 1 shows that the uncertainty contribution of \( Q \) is generally horizontally and linearly related to the canal section length for both \( m \) values. However, the uncertainty contribution of \( y \) increased sharply when the canal section was less than around 2200 m. When \( m \) doubled, the uncertainty contribution of \( Q \) decreased by almost 1/3. For canal sections shorter than 1000 m, the uncertainty contribution of \( Q \) was already lower than those of \( y_{1} \) and \( y_{2} \). The curves of \( y_{1} \) and \( y_{2} \) almost overlap, as shown in Figure 1, which implies the uncertainty contribution of \( y_{1} \) and \( y_{2} \) are nearly identical.

The relative uncertainty curves in Figure 2 show similar shapes to the curves of the \( y \) uncertainty contribution in Figure 1. The turning points of the curves both appeared at 2200 m on the x-axis. The relative uncertainty dropped by about 1/3 when \( m \) was doubled. Evidently, defining a canal section that is sufficiently long, which was at least 2200 m in the case of the MRP, is sufficient to achieve a relatively low uncertainty of \( n \).
5 CONCLUSIONS

Uncertainty analysis of Manning’s roughness coefficient measurements was conducted for 7 canal sections of the MRP. Error propagation rules were established, and the combined standard uncertainty formula of \( n \) was derived with the assumed independent variables of the discharge and water levels of the upstream and downstream measurement sites. The discharge uncertainty was calculated using a velocity-area method obtained from the ISO 748 standard. The main results are as follows:

a) All of the canal sections had nearly the same levels of uncertainty of the roughness coefficient measurement, which was around 0.002 for \( U_{95} \) and 13% for the relative uncertainty.

b) The discharge measurements were the main contributor to the roughness coefficient uncertainty compared with the upstream and downstream water level measurements, which were nearly identical. Therefore, priority should be given to improving the precision of the discharge measurement instrument to reduce the uncertainty.

c) The number of verticals (i.e., \( m \)) in the velocity-area method had a much greater weight than other sources of the discharge measurement uncertainty. By doubling \( m \) from 15 to 30, the relative uncertainty dropped by nearly 1/3.

d) The relationships between those uncertainty components can be useful for designing tests to achieve the expected levels of uncertainty for the roughness coefficients. In the MRP, setting the length of the canal section to be longer than 2200 m was sufficient for keeping a relatively low uncertainty of the \( n \) measurements.

It should be noted that values of the uncertainty components of the discharge measurements in the ISO 748 standard were derived from empirical studies. They provide a simple and robust, but not versatile and flawless (Despax et al., 2016), approach for conducting uncertainty analysis. Further study is still required to develop a more practical tool and to plan and improve the measurement strategy.

ACKNOWLEDGEMENTS

This work is supported financially by the Shenzhen Project for the comprehensive management of the Two River Basin (Longgang & Ping Shan River), the Shenzhen Project for comprehensive renovation and water quality improvement of the main stream of the Ping Shan River (CSCEC-PSH-2017-05). It is also supported by the Major Science and Technology Program for Water Pollution Control and Treatment of China (2017ZX07108-001), the National Natural Science Foundation of China (51579251).
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