

A HIGHLY EFFICIENT GENE EXPRESSION PROGRAMMING FOR VELOCITY DISTRIBUTION AT COMPOUND SEWER CHANNEL

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ABSTRACT

Flow velocity distribution offers key information in open channels studies, and accurate evaluations of it prepare a wide range of applications in different fields such as sediment transport and power plant design. In this study, the aptitude of a new approach based on gene expression programming (GEP) is perused in velocity field predicting of the compound open channel. A simple GEP-based model is proposed for velocity distribution estimation in an entire cross section of the channel. The proposed model is trained in eight different hydraulic conditions, and the generalized ability of this model are evaluated in three different condition, which is no role in velocity distribution modelling. The results show that the proposed model predicts the velocity distribution precisely ($MAPE=6.39$ & $RMSE=0.059$). Also, the results indicated the model excellently estimated the dip phenomena, which is a complex feature in velocity distribution prediction. Furthermore, a partial derivative sensitivity analysis is presented to survey the effect of each effective parameters in the proposed model.

Keywords: dip phenomena, field data, flow velocity profile, Gene Expression programming (GEP), compound sewer channel.

1 INTRODUCTION

The velocity dip is an intrinsic feature of narrow open channel flows, and it describes the phenomenon that the maximum stream wise velocity occurs below the flow surface. This feature caused by secondary currents mixing low and high momentum water by convection from the side-walls to water surface centre (Nezu and Nakagawa, 1993). The dip-phenomenon, including the corresponding cross-sectional velocity distribution, remains unsolved even for smooth rectangular open channel flow, a benchmark for studying river hydraulics and sediment transport. The dip phenomenon was reported more than a century ago by Stearns (1883) and Murphy (1904). Near the sidewall, the dip phenomenon also appears even if the aspect ratio is large enough. It is a challenge to scientists and engineers to describe and propose mathematical models to predict the velocity fields in open channels.

The essential processes of the velocity dip indicate that the influencing factors of the location of the maximum velocity with respect to the water depth include the sidewall roughness (the absolute sidewall effect), the distance to the sidewall (the absolute sidewall effect), the aspect ratio (the relative sidewall effect), the bed roughness, and free surface atmospheric conditions (Nezu and Nakagawa, 1993). Many researchers investigated the mechanism of the dip phenomenon in open channel flow and its effects on velocity distribution in narrow channels and proposed some mathematical equations (Yang et al., 2004; Maghrebi and Rahimpour, 2005; Bonakdari et al., 2008; Pu et al., 2010; Lassabatere et al., 2013; Bonakdari and Moazamnia, 2015). However, their study showed limitations in reflecting the hydraulic behavior of open channels, but their results indicated this method could predict the velocity distribution in narrow channels reasonably well, so it had to be expanded for practical application.

On account of the high performance of artificial intelligence (AI) based techniques in complex nonlinear problems results in to their extensive utilize in hydraulic engineering (Sattar et al., 2015; Ebtehaj and Bonakdari, 2016; Azimi et al., 2016; Sharafi et al., 2016; Sattar et al., 2017; Golami et al., 2018a,b; Ebtehaj et al., 2018; Milukow et al., 2019; Power et al., 2019; Ebtehaj et al., 2019). Most of these methods have a black-box and present the predicted values without a certain expression to use in a future application. One of the AI-based method, which is present a practical mode is Gene Expression Programing (GEP). Ebtehaj et al. (2015a) used GEP in the prediction of discharge coefficient in side weir. The authors used two different datasets for training and testing (separately) to validate the GEP-based model in terms of different hydraulic condition with modeling condition. The results indicated the proposed GEP model performs better than existing equations in term of discharge coefficient prediction. To predict the mean velocity in open channel intake, Karimi et al. (2016)

employed GEP. They present an equation which has a high level of accuracy flow mean velocity predicting in different widths of the intake channel. Recently, Bonakdari et al. (2018) developed an extreme learning machine approach to predecit velocity filed in sewers. But, the critical point with its application is selecting the correct number of hidden layer neurons, otherwise unacceptable results are achieved. Based on the authors' knowledge, the velocity distribution prediction at compound sewers by GEP-based techniques are not used in literature. Therefore, GEP technique is employed to predict velocity flow for the first time in this study. The main aim of this study is to present a new simple model based on GEP to predict velocity distribution in compound sewer channels. The channel coordinate and flow discharge are employed as GEP model inputs to predict velocity at any point of the channel. The used data in GEP development are collected from real-scale site within a sewer collection network in Nantes, France. The proposed GEP-based model are evaluated by three flow depths which are not used in model training, and two of which are the lowest and highest depth that not considered in training, are used. Also, partial derivative sensitivity analysis is employed to study the changing trend of each input in velocity distribution prediction.

2 FIELD DATASET

The used field dataset in this study is collected from Bonakdari (2006), which are done in Cordon Bleu on the main sewer line in Nantes, France. The channel with a bank or walkway in the left side of flow direction and has a compound oval, narrow cross section ($1.7 < Y/W < 2.6$; where Y and W are a horizontal and vertical axis, respectively). The flow distribution is measurement at 9 different discharges ($0.54 < Q \text{ (m}^3/\text{s)} < 1.65$) and 10 different depth ($0.65 < D \text{ (m)} < 1.19$) by a two-dimensional remote control device namely as Cerbere (Larrarte and Cattineau, 2005). The flow was subcritical (Froude number in the range of 0.2-0.3) and turbulent (Reynolds number greater than 10^5) with 0.49 m/s and 0.97 m/s as lowest and highest velocity, respectively.

To predict velocity distribution by GEP, the 7 different depths with 363 samples are selected for training, and the rest of data (166 samples) were are related to three depths with lowest and highest depths ($D = 0.65, 0.91, 1.19 \text{ m}$) are separated for validation the GEP-based modeling. Figure 1 presents the cross section and measurement points.

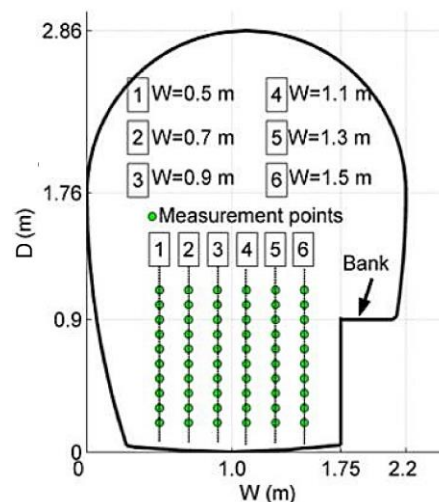


Figure 1: Scheme of the experimental site and measurement points.

3 OVERVIEW OF GEP

Genetic Programming (GP) is a supervised machine learning method that explorations a computer space. The standard GP evolve computer programs by symbolic regression. If the traditional methods are fairly intricate or specific mathematical physics-based models do not exist, the GP techniques could be considerably suitable. Ferreira (2001) introduced Gene Expression Programming (GEP), which is a nature expansion of GP, which is different from GP because of the cost of managing tree structure. GEP is an evolutionary intelligent models which is inspired of Darwinian evolution theory. This technique used the advantage of the previous version of genetic-based methods (Genetic Algorithm (GA) and GP), simultaneously. GEP is a phenotype-genotype algorithm which is employed the abilities and simplicity of GP and GA, respectively and overcome limitations of these genetic-based techniques. Like the GP and GA, the GEP modeling is started by a random initial population and applied genetic operators to reach the optimum results. The genomes in this technique are as linear symbolic strings similar to GP that follows Karva language to show the phenom as expression trees (ETs). The Karva is an introduced language by Ferreira (2001) for coding programs in each chromosome.

The GEP chromosomes have functions and terminals which are must be defined before starting the modeling process. The GEP is consist of two objects (i.e., chromosomes and ETs) so that chromosomes can

be include of one or more genes or sub-expressions. Each gene is contained of two sections, head, and tail. A head could be consist of function and terminal symbols while a tail has only terminal symbols. The head length (h) must be determined using trial and error by the user while tail length (t) is a function of the maximum argument of the function (n_{max}) and head length and equal to $h(n_{max}-1)+1$.

Two strong points of GEP is a simple genetic diversity and multi genetic nature. The first one is performed by different genetic operators, which are more than those that are presented in GP at the chromosome level, and the other one leads to more complex programs consist of several sub-programs. Therefore, GEP is outstripped GP in 100-100000 times (Ferreira, 2001). However, the advantages of GEP are obvious from its nature. The most important ones are (Ferreira, 2002): a) Simple entities of chromosomes: linear, relatively small, compact and easy to manipulate genetically (mutate, replicate, recombine, etc.) b) the ETs are absolutely the presentation of their corresponding chromosomes and selected due to their fitness to reproduce with modification in new generation.

The presented genetic operators in GEP is more than GP which have different roles. The main operators that applied to chromosomes are transposition, mutation, and recombination. The most efficacious genetic operator with intrinsic modification power is a mutation, which can happen anywhere in the chromosome without changing the chromosome length. In the chromosome head, the function or terminals can be substituted with each one while in the chromosome tail, terminals are only substituted with terminals.

In the first step of GEP modeling, a random initial population is produced, and chromosomes are expressed. These encoded programs are presented as ETs. Then, the fitness of those is calculated and check the termination condition. If the termination condition is satisfied, the presented ETs are selected as final results. Otherwise, the individuals are selected by roulette wheel selection method and using genetic operators (i.e., mutation, transposition, and recombination), the reproduction is done for new generations and computes the fitness of new generations. This process is continued until the termination criteria are satisfied.

4 DEVELOPMENT OF GEP IN VELOCITY FIELD PREDICTION

The procedure of streamflow forecasting is as follows: The first step is the selection of fitness function. For this problem, the root relative squared error ($RRSE$) fitness function of an individual program, i , was applied (Ferreira, 2006):

The velocity field modeling process using GEP is started by a selection of fitness function, which is evaluating the modeling results during the training stage. The root relative squared error ($RRSE$) fitness function of individual program i , (Ferreira, 2006) is employed in velocity field prediction, which is defined as follows:

$$RRSE_i = \sqrt{\frac{\sum_{j=1}^n (P_{ij} - T_j)^2}{\sum_{j=1}^n (T_j - \bar{T})^2}} \quad [1]$$

where P_{ij} and T_j are the predicted value by the individual program i and target value (respectively) for fitness case j and \bar{T} is average of target value fitness case j , which are calculated as follows:

$$\bar{T}_j = \frac{1}{n} \sum_{j=1}^n T_j \quad [2]$$

The best results are gained in $RRSE_i=0$. Because of that, the $RRSE_i$ does not increase with program performance increasing; the $RRSE_i$ term could not be used as fitness function straightly. Therefore, the following equation is defined to evaluate the fitness of an individual program (f_i):

$$f_i = \frac{1000}{1 + RRSE_i} \quad [3]$$

The f_i range is between 0 and 1000 so that the best fit is reached at f_i , which has $RRSE_i=0$.

After selection of fitness function in the first step, the function sets (F) and terminals set (T) should be select at the second step to generate different chromosomes. In the current study, the terminals sets which are input variables are consist of flow discharge (Q) and coordinates parameters (D , W , Y) so that $T=\{D, W, Y, Q\}$, where D , W , and Y are depth, vertical axis and horizontal axis. The selection of function set does not require a certain method and depends on the standpoint of the user. In this study, first, the modeling is started by considering the four arithmetic operation $\{+, -, \times, /\}$ as a function set. In continue, trigonometric functions (\sin , \cos , \tan , \cot) and corresponds hyperbolic trigonometric functions were also added individually. Also, the $\{x^2$, x^3 , \sqrt{x} , and $3Rt\}$ are also individually in different GEP modeling to predict velocity distribution. By considering the simplicity and

accuracy of different models, the $\{+, -, \times, /, \sin, \sqrt{}, 3Rt, \ln, x^3\}$ were considered as function set. The architect of chromosomes (i.e., the head length and the number of chromosomes) are chooses in the third step. The length head and the number of chromosomes was consider different from 5 to 12 and 30 to 100 by Ferreira (2006) suggestion. The different combinations of head length and number of chromosomes are applied for GEP modeling. The best models by considering the accuracy and simplicity simultaneously are selected with 30 chromosomes and $h=8$. In step four, the number of genes and linking functions are chooses. The linking function must be chosen as “addition”, “division”, “subtraction” and “multiplication” for algebraic sub trees. The number of genes are selected as 3 by trial and error and the sub trees are linked by “Addition”. The lower value for genes could not present an accurate GEP model and also the higher number of it results in a high complex model. In the final step, the value of genetic operators should be adjusted. The parameters’ value per runs are as: inversion rate: 0.15, mutation rate: 0.014, gene transposition rate: 0.15, gene recombination rate: 0.15, one-point recombination rate: 0.35, two-point recombination rate: 0.35, root insertion sequence transposition: 0.1, insertion sequence transposition rate: 0.1.

The final optimum GEP-based model in prediction of velocity distribution is presented as follows:

$$U = \left[\left(\sin(1.7W) \right)^{0.25} \times \sin(Y^{1/6}) \right] + \left[(Y \times W - D)(Y - D) \right]^{1/3} + 0.145 \left[\ln(Q) - Y^2 + D \right] \quad [4]$$

5 RESULTS AND DISCUSSION

In this study, in order to check the accuracy of the presented model for predicting of velocity distribution at compound sewer channels by GEP, two relative and absolute statistical indices namely as mean absolute percentage error (MAPE) and root mean square error (RMSE), respectively which are defined as follows:

$$MAPE = \frac{1}{n} \sum_{i=1}^n \left(\frac{|x_i - y_i|}{x_i} \right) \times 100 \quad [5]$$

$$RMSE = \frac{1}{n} \sum_{i=1}^n (x_i - y_i)^2 \times 100 \quad [6]$$

Figure 2 survey the performance of GEP-based modeling in predicting of velocity distribution in different depths ($D=0.65$, $D=0.91$; $D=1.19$ m) which are not used in training and consider the lowest and highest depth among all 7 depths. This figure shows that the presented model at all depths is intended to test the model, the performance is well, so that most of the values predicted by the GEP, has a relative error less than 10%. The results of the predicted velocity in $D = 0.65$ shows that most estimable values are overestimated. The average relative error at $D = 0.91$ ($MAPE = 2.783$ $RMSE = 0.03$) and $D = 1.19$ ($MAPE = 3.307$ $RMSE = 0.0305$) respectively are, less than 3% and 4%.

So according to the provided description, generally function well in predicting the velocity distribution, but the physical distribution process on different widths for the velocity distribution at the channel are also must be evaluated. Hence the figure 3, 4 and 5, the anticipated velocity performance within three different channel depth for different values of the width are compared with the measurement values in Cordon Bleu channel.

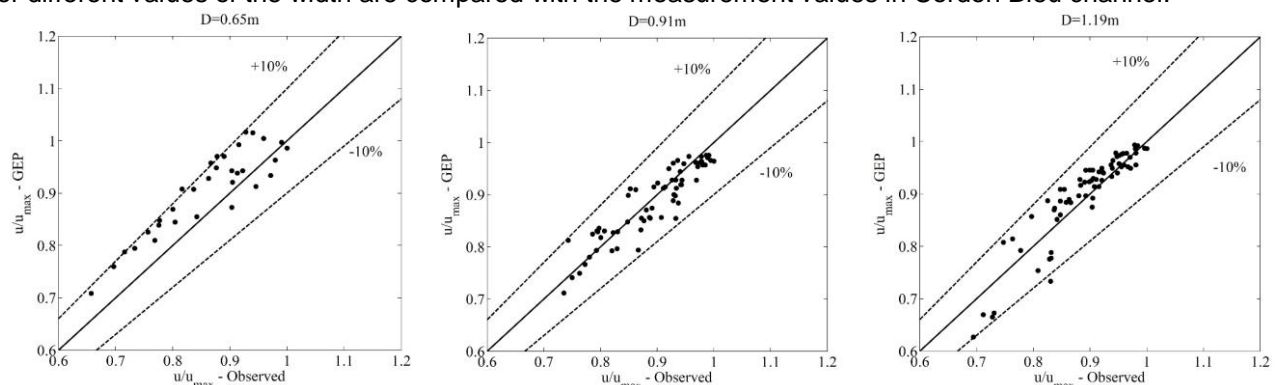


Figure 2. Evaluation of GEP in the prediction of the velocity distribution

Figure 3 presented velocity distribution in compound sewer channel at a depth of 0.65 m for 6 different widths are presented in Figure 1. The intended width in Figure 1, relative to the center of the channel, are symmetrical. The results showed that for close to the center of the channel ($W = 0.7, 0.9, 1.1, 1.3$ m) get results of GEP-based model of bulging precision is very high so that the estimated amounts and measured values are good adopted. In addition, the model proposed in this study, the dip phenomenon, as well in this width, are anticipated. For close to the wall, two different trends are presented. The width of the closest to the bank ($W = 1.5$) in addition

to the low modeling error, of good accuracy in predicting the dip phenomenon but for $W = 0.5$, the offered model does not provide a good flow profile, though it predicted a low difference values with the measured values.

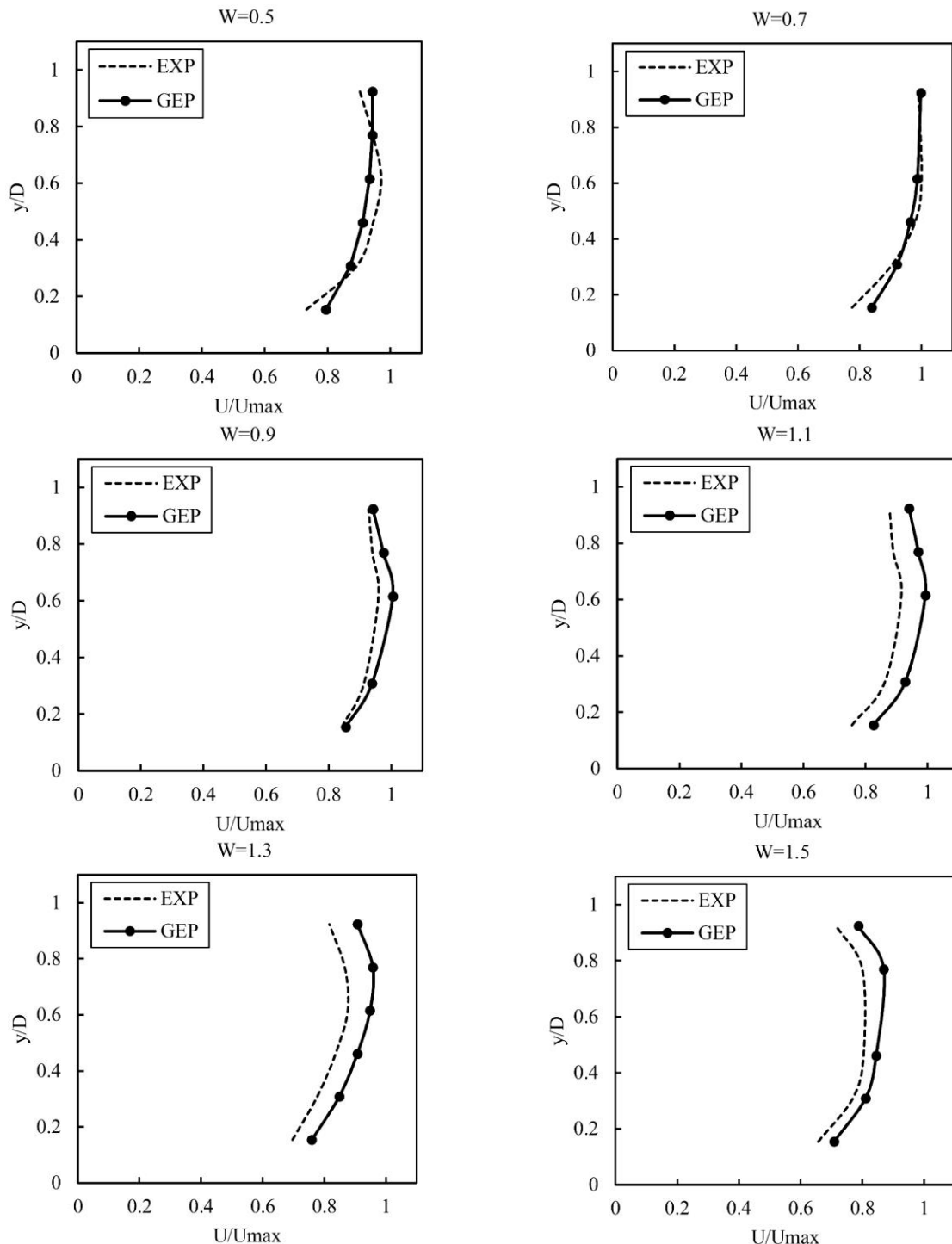


Figure 3. Velocity distribution based on GEP model for $D=0.65$ m

Figure 4 indicates the velocity distribution in $D = 0.91$ m. The predicted values using GEP shows that for the width away from the bank, the accuracy of modeling is very good, and the GEP has been able to predict dip phenomenon. This trend is for the wide close to the center of the channel ($D = 0.9$ & 1.1 m) is also similar, but it approached a bank, the modeling precision is decreased. However, according to the presented indices values in Figure 2, the average relative error for this depth, are less than 3%, which represents a fairly good estimate of velocity distribution in compound sewer channel.

Figure 5 demonstrated the anticipated results of the velocity distribution based on the proposed model at $D = 1.19$, in 6 different widths. According to the figure velocity distribution in compound channel has been well

anticipated so that, except for the width $W = 1.5$ m, in addition to high precision of velocity prediction, velocity distribution modeling process similar to the actual process is also provided on the channel, and the maximum velocity and dip phenomenon is also well demonstrated. For the nearest width to bank or walkway, the model offered compared to the other widths, has less accuracy in forecasting the velocity distribution.

Due to the provided description, the offered model based on GEP predicted the velocity distribution in a compound open channel with high accuracy of the circumstances that most of the projected values rated less than 10% of the error and the maximum average relative error of about 7%.

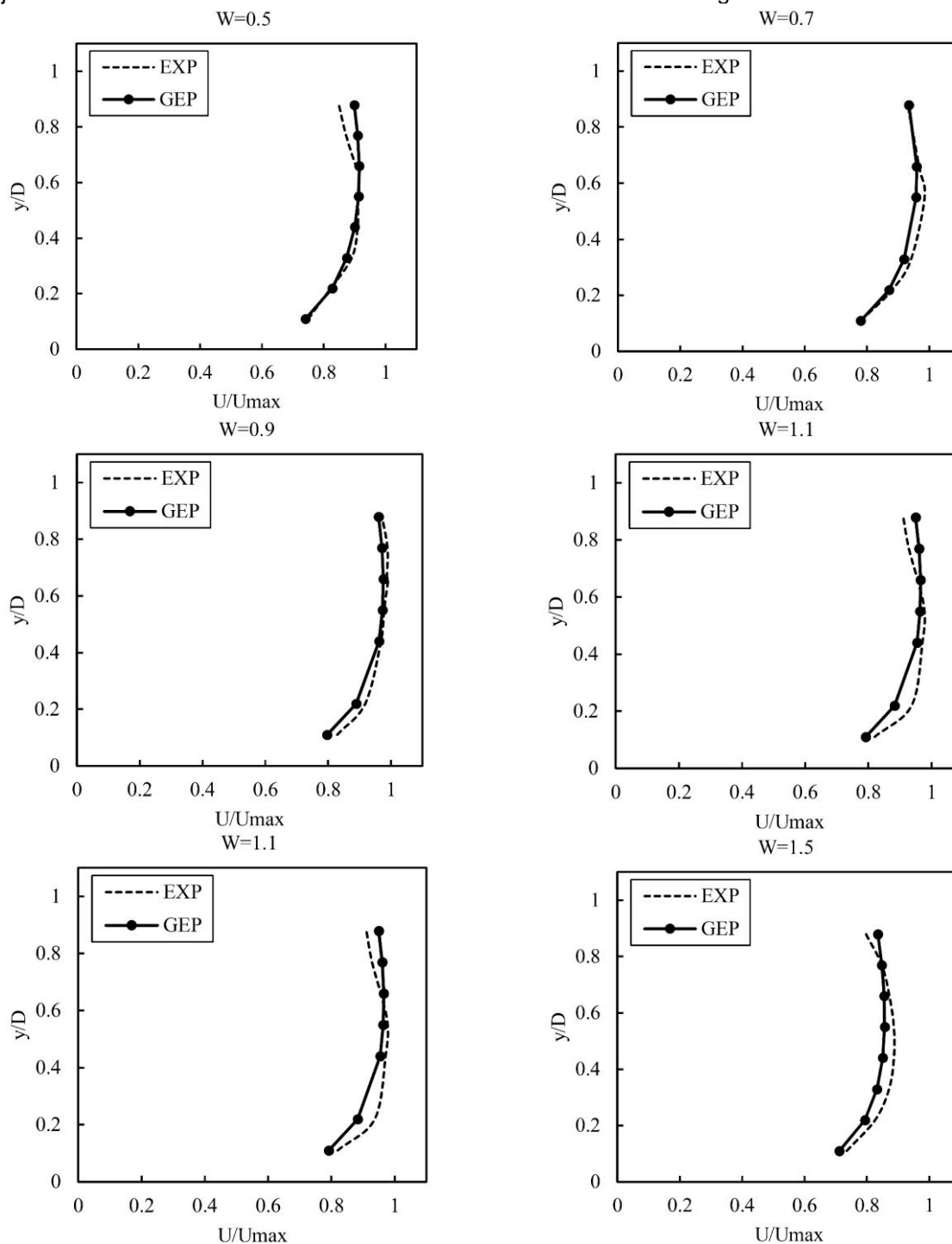


Figure 4. Velocity distribution based on GEP model for $D=0.91$ m

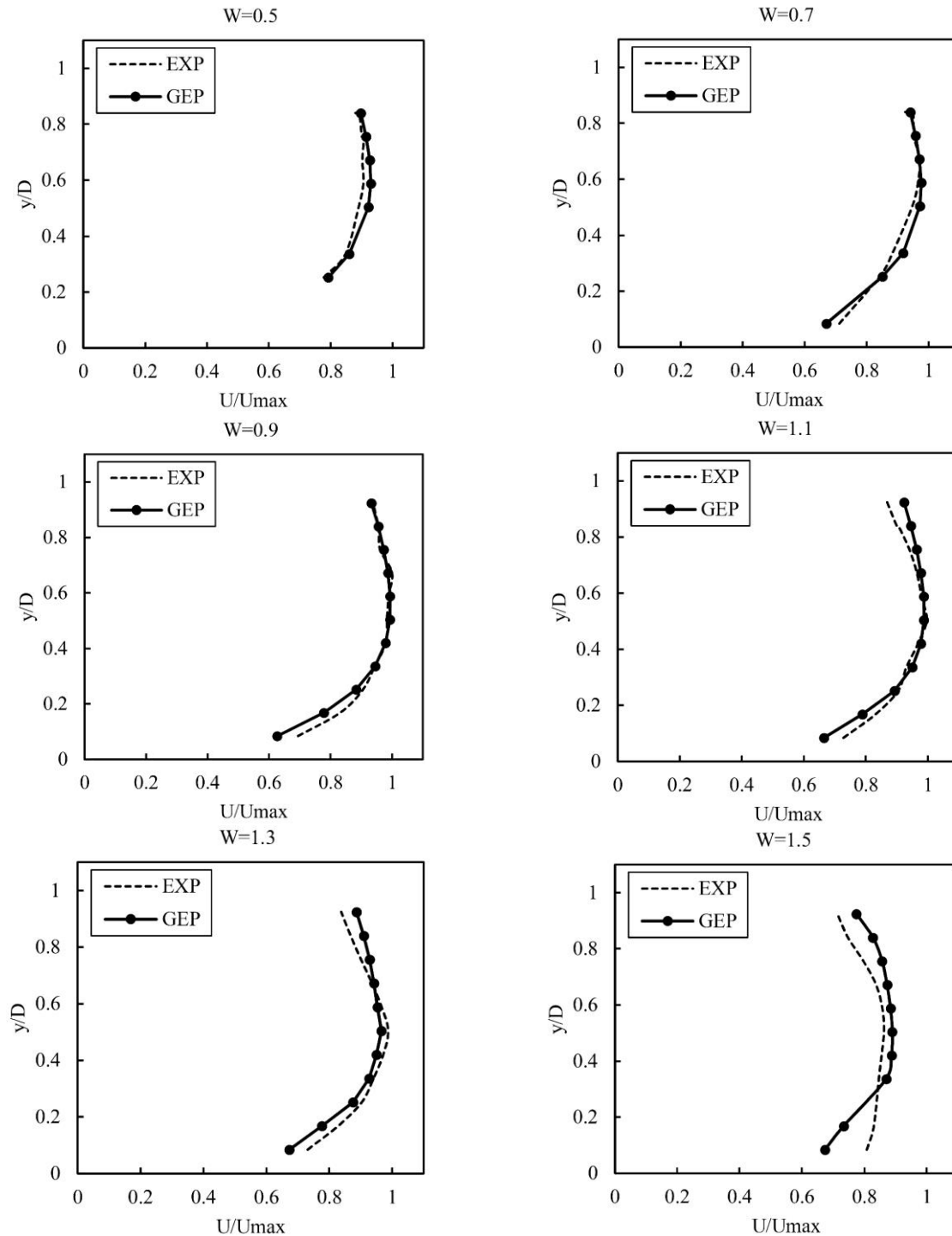


Figure 5. Velocity distribution based on GEP model for $D=1.19$ m

In this section, the changing trend of the flow velocity in compound sewer channels related to used input variables in Eq. (4) to estimate flow velocity is reviewed and evaluated. Therefore, the partial derivative sensitivity analysis (PDSA) method (Ebtehaj et al., 2015b) utilized. On the PDSA method, the sensitivity of dependent target variable (U) related to each independent variables (i.e., D , Z , Y & Q) is surveyed by partial derivative. The relationship between the amount of sensitivity achieved and a variable which is derivative is done is direct so that the increase (or decrease) of sensitivity indicates an increase (or decrease) the desired parameter efficiency relative to the parameter target (i.e., U in this study) if the value of the partial derivative is positive (or negative), the increase in the value of the input parameter (i.e., D , Z , Y & Q), leading to an increase (or decrease) the value of the parameter target (i.e., U).

Figure 6 shows the partial derivative of the sensitivity analysis of Eq. (4) related to input variables in this equation. The sensitivity of the relationship proposed in this study relative to the variable D is presented both

positive and negative which are exactly cannot be stating that increase or decrease the value of this variable, that direct or indirect impact on the flow velocity. It is noteworthy that for values of $D > 0.7$, increase the amount of D , reduce the amount of U . The sensitivity of the variable Z shows that by increasing the value of this variable, the sign of sensitivity values are changed from positive to negative. As for $Z < 1$, increase the amount of this variable, increasing from U . The variable Y has the same process also has D and Z , and the sensitivity of this variable is not constant so that for $Y < 0.6$, the increase of these variables results in increasing U value. Unlike another variable, the variable Q has a positive sensitivity as an increase (or decrease) in Q leads to U increasing (or decreasing). It should not be that the increasing Q led to decreasing sensitivity so that the highest increasing value of U is reached at the lowest value of Q .

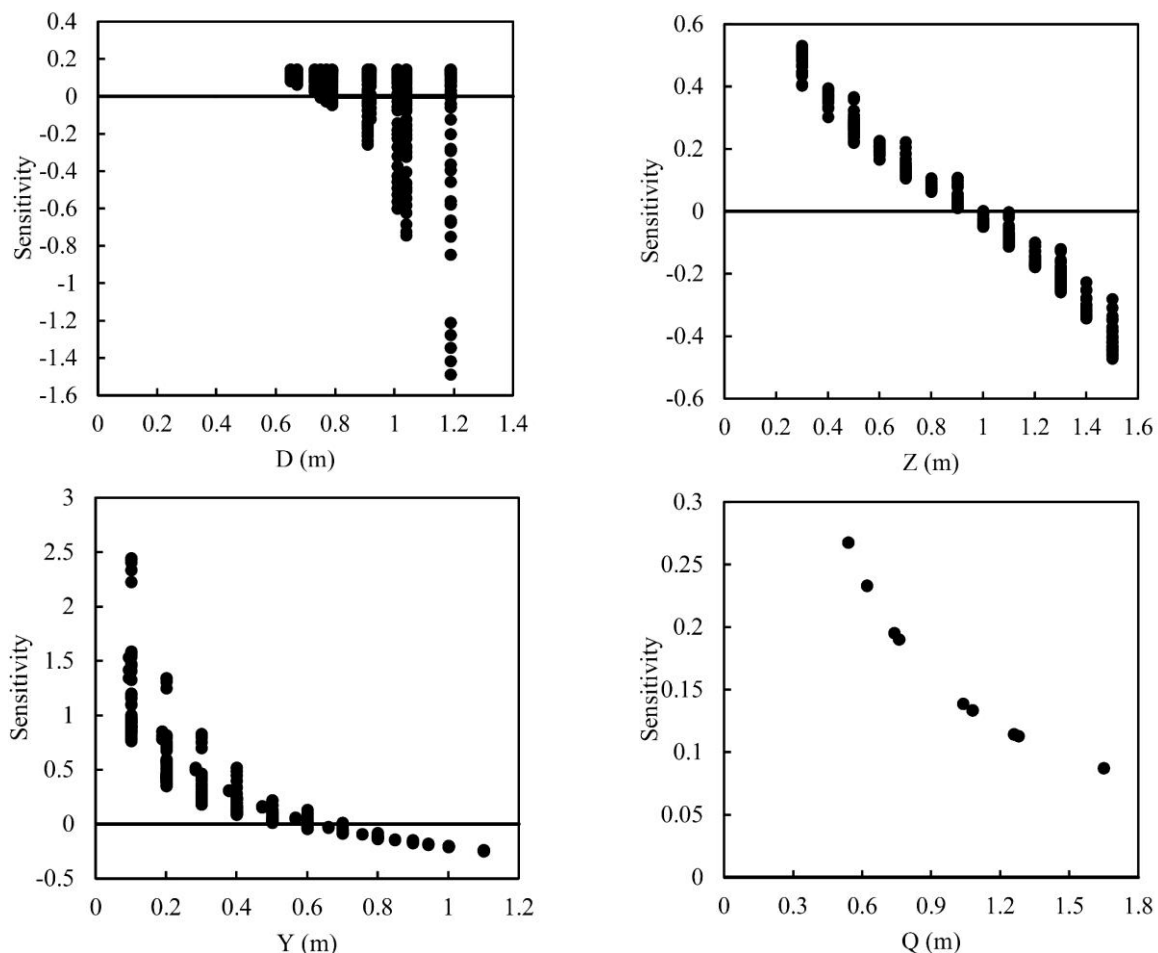


Figure 6. Partial derivative sensitivity analysis result of different input variables in GEP-based modeling of the velocity distribution

6 Conclusions

In this study, a new model is presented to predict velocity distribution in compound sewer channel at different hydraulic conditions. The presented method is simple and predicts velocity distribution in each point with different width and depth. The presented model validation was done with three different hydraulic conditions, which are different with training condition. For all flow depths ($D = 0.65, 0.91, 1.19$ m) the GEP model often estimated velocity distribution with less than 10% relative error so that $D=0.65$ ($MAPE=6.39$; $RMSE=0.059$), $D=0.91$ ($MAPE=2.78$; $RMSE=0.03$) and $D=1.19$ ($MAPE=3.307$; $RMSE=0.03$). The weakest performance of GEP-based model in velocity distribution predicting is related to cross section closest to the bank where velocity was measured ($W = 1.5$). But, it should not be that most of predicted value has relative error lower than 10% and could be predicted dip phenomena. Also, a partial derivative sensitivity analysis is used to study the effect of each input variables in the prediction of velocity distribution. The results show that coordinate number don't have a constant trend in velocity distribution prediction while the flow discharge has a direct relation with velocity distribution so that the increase (or decrease) of this variable lead to increase (or decrease) of velocity predicted value by GEP. Based on the relatively good performance of the proposed model based on GEP, this model could be used as an alternative model instead of existing ones.

REFERENCES

- Azimi, H., Bonakdari, H., Ebtehaj, I. and Michelson, D.G.A. (2016). Combined adaptive neuro-fuzzy inference system–firefly algorithm model for predicting the roller length of a hydraulic jump on a rough channel bed. *Neural Computing and Applications*, 1-10. doi:10.1007/s00521-016-2560-9
- Bonakdari, H. (2006). Modelisation des écoulements éconlecteur dassainissement-application a la conception de points de mesures. PhD Thesis, University of Caen, Caen, France.
- Bonakdari, H., Larrarte, F., Lassabatere, L. and Joannis, C. (2008). Turbulent velocity profile in fully-developed open channel flows. *Environmental Fluid Mechanics*, 8 (1), 1-17.
- Bonakdari, H. and Moazamnia, M. (2015). Modeling of velocity fields by the entropy concept in narrow open channels. *KSCE Journal of Civil Engineering*, 19 (3), 779-789.
- Bonakdari, H., Zaji, A.H., Gharabaghi, B., Ebtehaj, I. and Moazamnia, M. (2018). More accurate prediction of the complex velocity field in sewers based on uncertainty analysis using extreme learning machine technique, *ISH Journal of Hydraulic Engineering*, 1-12, <https://doi.org/10.1080/09715010.2018.1498753>
- Ebtehaj, I., Bonakdari, H. and Gharabaghi, B. (2018). Development of more accurate discharge coefficient prediction equations for rectangular side weirs using adaptive neuro-fuzzy inference system and generalized group method of data handling. *Measurement*, 116, 473-482.
- Ebtehaj, I. and Bonakdari, H. (2016). Assessment of evolutionary algorithms in predicting non-deposition sediment transport. *Urban Water Journal*, 13(5), 499-510.
- Ebtehaj, I., Bonakdari, H., Zaji A.H., Azimi, H. and Sharifi, A. (2015a). Gene expression programming to predict the discharge coefficient in rectangular side weirs. *Applied Soft Computing*, 35, 618-628.
- Ebtehaj, I., Bonakdari, H., Zaji, A.H., Azimi, H. and Khoshbin, F. (2015b). GMDH-type neural network approach for modeling the discharge coefficient of rectangular sharp-crested side weirs. *Engineering Science and Technology, an International Journal*, 18 (4), 746-757.
- Ebtehaj, I., Bonakdari, H. and Gharabaghi, B. (2019). Closure to “An integrated framework of Extreme Learning Machines for predicting scour at pile groups in clear water condition by Ebtehaj, I., Bonakdari, H., Moradi, F., Gharabaghi, B., Khozani, Z.S.” *Coastal Engineering*, 147, 135-137.
- Ferreira, C. (2001). Gene expression programming: a new adaptive algorithm for solving problems. *Complex Systems*, 13 (2), 87–129
- Ferreira, C. (2002). Gene expression programming in problem solving. Part VI, In: *Soft Computing and Industry*, 635–653.
- Ferreira, C. (2006). Gene expression programming: mathematical modeling by an artificial intelligence, 2nd ed., Berlin-Heidelberg, Springer-Verlag.
- Gholami, A., Bonakdari, H., Ebtehaj, I., Mohammadian, M., Gharabaghi, B. and Khodashenas, S.R. (2018a). Uncertainty analysis of intelligent model of hybrid genetic algorithm and particle swarm optimization with ANFIS to predict threshold bank profile shape based on digital laser approach sensing. *Measurement*, 121, 294-303.
- Gholami, A., Bonakdari, H., Zeynoddin, M., Ebtehaj, I., Gharabaghi, B. and Khodashenas, S.R. (2018b). Reliable method of determining stable threshold channel shape using experimental and gene expression programming techniques. *Neural Computing and Applications*, 1-19. DOI: 10.1007/s00521-018-3411-7.
- Karimi, S., Bonakdari, H., Ebtehaj, I. and Zaji, A.H. (2016). Prediction of Mean Velocity in Open Channel Intake Using Numerical Model and Gene Expression Programming. *Mitteilungen Saechsischer Entomologen (MSE)*, 119, 599-616.
- Larrarte, F. and Cattineau, L.M. (2005). Cattineau Nouveaux instruments pour connaître les champs de vitesses et de concentrations. *Revue européenne de génie civil*, 9 (7-8), 963-978.
- Lassabatere, L., Pu, J. H., Bonakdari, H., Joannis, C. and Larrarte, F. (2013). Velocity distribution in open channel flows: Analytical approach for the outer region. *Journal of Hydraulic Engineering*, 139 (1), 37-43.
- Maghrebi, M.F. and Rahimpour, M. (2005). A simple model for estimation of dimensionless isovel contours in open channels. *Flow Measurement & Instrumentation*, 16 (6), 347-352.
- Milukow, H.A., Binns, A.D., Adamowski, J., Bonakdari, H. and Gharabaghi, B. (2019). Estimation of the Darcy–Weisbach friction factor for ungauged streams using Gene Expression Programming and Extreme Learning Machines. *Journal of Hydrology*, 568, 311-321.
- Murphy, C. (1904). Accuracy of stream measurements. *Water Supply and Irrigation*, 95, 111–112.
- Nezu, I. and Nakagawa, H. (1993). Turbulence in Open-channel Flows. Lisse: Balkema Publishers.
- Power, H.E., Gharabaghi, B., Bonakdari, H., Robertson, B., Atkinson, A.L. and Baldock, T.E. (2019). Prediction of wave runup on beaches using Gene-Expression Programming and empirical relationships. *Coastal Engineering*, 144, 47-61.
- Pu, J.H., Bonakdari, H., Lassabatère, L., Joannis, C. and Larrarte, F. (2010). Turbulent velocity profiles: A new law for narrow channels. *Houille Blanche*, 3, 65-70
- Sattar, A.M.A., Gharabaghi, B., Sabouri, F. and Thompson, A.M. (2017). Urban stormwater thermal gene expression models for protection of sensitive receiving streams. *Hydrological processes*, 31(13), 2330-2348.

- Sattar, A.M. and Gharabaghi, B. (2015). Gene expression models for prediction of longitudinal dispersion coefficient in streams. *Journal of Hydrology*, 524, 587-596.
- Sharafi, H., Ebtehaj, I., Bonakdari, H. and Zaji, A.H. (2016). Design of a support vector machine with different kernel functions to predict scour depth around bridge piers. *Natural Hazards*, 84 (3), 2145-2162.
- Stearns, F.P. (1883). On the current meter, together with a reason why the maximum velocity of water flowing in open channel is below the surface. *Trans. ASCE*, 7, 331–338.
- Yang, S.Q., Tan, S.K. and Lim, S.Y. (2004). Velocity distribution and dip phenomenon in smooth uniform open channel flow. *Journal of Hydraulic Engineering*, 130 (12), 1179-1186.