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LEAK LOCALIZATION USING TIME REVERSAL TECHNIQUE

MUHAMMAD WAQAR(1), MOEZ LOUATI(2), & MOHAMED S GHIDAOUI(3)

(1,2,3) Department of Civil and Environmental Engineering, The Hong Kong University of Science and Technology, Hong Kong
mwaqar@ust.hk; mlouati@ust.hk; ghidaoui@ust.hk

ABSTRACT

Time-reversal technique together with the analytical solution for the 1D wave equation is proposed for pipeline leak localization. The analytical solution is derived for an unbounded pipe system wherein the dynamic response of the boundaries is not incorporated. The basic principle of the technique is to reverse the measured impulse response function (IRF) in time and convolve with the analytical IRF. The output signal of this convolution will only be maximum when the two response functions contain the same information of the leak location. In the frequency domain, this convolution integral becomes the complex conjugate of the measured IRF multiplied with the analytical IRF. This product will also attain its maximum when the frequency content in both signals are in phase. Both kinds of information are utilized to localize the leaks in time and frequency domain. The purposed method is tested numerically for one leak case in the presence of measurement noise. The leak is successfully localized.

Keywords: Pipeline defects, leak detection, time-reversal, hydraulic transients, water distribution systems

1 INTRODUCTION

Leakage or defect detection in freshwater pipelines is one of the recent and emerging applications of hydraulic transients. Flow control devices (e.g. in-line valves or pumps) are used as a source to generate pressure (acoustic) wave(s) inside the pipe in a controlled environment. As the wave propagates, it senses the pressure difference caused by the defect and partially gets reflected (Colombo et al. 2009). These reflections are measured at several locations along the pipe, known as the pressure-head response of the system, and used to identify and locate the anomalies (e.g. leaks, blockages) using different methodologies. A comprehensive review of these methods can be found in Colombo et al. (2009) and Abdulshaheed et al. (2017). Problems arise when the measurements are masked by the inevitable measurement noise, which is often the case, and the signal-to-noise ratio (SNR) is also small. Under such situations, many of the existing methods in the foregoing literature either do not work or produce inaccurate results (Wang and Ghidaoui 2018).

Recently, a promising and noise-tolerant signal processing method called matched-field processing (MFP) is recently introduced for pipeline leak detection (Wang and Ghidaoui 2018, Wang et al. 2019). The method is applied in frequency domain using transfer matrix model which allows to make the estimation of leak locations and their sizes separately. However, there are few concerns which may limit its applicability in the field. First, it requires at least $2n+1$ (n = number of pipes) measurement locations to detect the leak(s) which may not always be fulfilled in the field. Second, it has the issue of getting trapped into local maxima (i.e. the side lobes) which is misleading, especially for the case when the number of leaks are unknown. This particular issue may become prominent for the case when the system has dynamic boundaries (e.g. pressure reducing valve or pump) which cannot be modeled accurately using transfer matrix. Therefore, a technique that is as noise-tolerant as MFP and requires fewer measurement locations to detect leaks without being trapped into local maximum is desired.

In essence, MFP makes use of the phase conjugate of the received signals in the frequency domain. Phase conjugate operation is equivalent to reversing (flipping) the signals in the time so that whatever comes last becomes first and vice versa (Tolstoy 1993). For wave problems, this time-reversed signals are re-emitted from the receivers and the wave refocuses back to its source, called wave refocusing (Fink 1992, de Mello et al. 2016). The key principle of this phenomena relies on the time-invariance property of the acoustic wave equation in a non-dissipative medium. To further elaborate, consider the one-dimensional acoustic wave equation as given by

$$\frac{\partial^2 h(x,t)}{\partial t^2} - a^2 \frac{\partial^2 h(x,t)}{\partial x^2} = 0 \quad [1]$$

where h is the pressure head response of the system, a is the wave speed, and x and t are the space and time coordinates, respectively. In this equation, the second-order time derivative operator allows the wave propagation backward in time (i.e. when $t=-t$) without changing the original form of the equation as

$$\frac{\partial^2 h(x,-t)}{\partial(-t)^2} - a^2 \frac{\partial^2 h(x,-t)}{\partial x^2} = 0. \qquad [2]$$

This means that, if $h(x,t)$ is the solution, $h(x,-t)$ is also the solution for Eq. [1]. Time-invariance property has been extensively utilized in many fields such as underwater acoustics (Kuperman et al. 1998, Hodgkiss et al. 1999, Fink et al. 2000), geophysics (Larmat et al. 2006, Larmat et al. 2010), medical imaging (Fink 1992, Wu et al. 1992, Fink et al. 2000, Montaldo et al. 2004) and structural health monitoring (Wang et al. 2004, Park et al. 2005, Park et al. 2007) for acoustic source localization. The present work aims to establish the concept and framework for time-reversal (TR) for pipeline leak localization. First, the paper provides an analytical solution of the transient wave equation which does not incorporates the dynamic response of the boundaries. Meaning, the solution only takes into account the reflections which arrive prior to the reflection from any known dynamic boundary. The analytical solution is then combined with TR technique to make localize the leak(s). The proposed method is first proved mathematically and then tested numerically. The future work and conclusions are drawn towards the end.

2 METHODOLOGY

2.1 Analytical model for the unbounded pipe system

Consider a viscoelasticity pipe system, the transient response of the system can be governed by the one-dimensional water hammer equations in the frequency domain as (Wang et al. 2019)

Mass conservation:
$$\frac{dq}{dx} + \left(\frac{gA}{a_{ve}^2}\right) i\omega h = 0 \qquad [3]$$

Momentum conservation:
$$\frac{dh}{dx} + \frac{1}{gA} i\omega q + Rq = 0 \qquad [4]$$

where,
$$a_{ve} = \left[\rho\left\{\frac{1}{K} + (1-\nu^2)\frac{D}{e}\left(J_o + \sum_{j=1}^{N_{kv}} \frac{J_j}{1+i\omega\tau_j}\right)\right\}\right]^{-1/2}, \qquad [5]$$

q and h are the flow and head perturbation terms; a_{ve} is the frequency-dependent wave speed in the viscoelastic pipe, g is the gravitational acceleration, A is the cross-sectional area of the pipe, ω is the angular frequency, $R = fQ_o/(gDA^2)$, f is the Darcy-Weisbach friction factor, Q_o is the steady-state flow rate, D is the pipe diameter, ν is the Poisson's ratio and J_j, τ_j and N_{kv} are the coefficients of the Kelvin-Voigt (KV) model to incorporate the viscoelastic effects of the pipe walls. Differentiating Eq. [3] with respect to x and subtracting from Eq. [4] results the one-dimensional wave equation as

1D wave equation:
$$\frac{d^2 q}{dx^2} + \left(\frac{\omega^2}{a_{ve}^2} - \frac{gA}{a_{ve}^2} i\omega R\right) q = 0. \qquad [6]$$

The analytical solution for Eq. [6] is assumed to have the form of

$$q(\omega,x) = I(\omega)\exp(-i\mu x). \qquad [7]$$

Substituting Eq. [7] in Eq. [6] and simplifying for μ results

$$\mu = \sqrt{\frac{\omega^2}{a_{ve}^2} - \frac{gA}{a_{ve}^2} i\omega R}. \qquad [8]$$

Consider a source is located at $x = x_s = 0$ in the form of a delta function $\delta(t)$ and a discrete defect is located at $x = x_L$ as shown in Figure 1. The defect will reflect part of the incident wave and transmit the rest which allows to separate the solution for reflected and transmitted wave as

For pipe 1: $$q(x) = 1\exp(-i\mu x) + R\exp(i\mu x) \quad [9]$$

For pipe 2: $$q(x) = T\exp(-i\mu x) \quad [10]$$

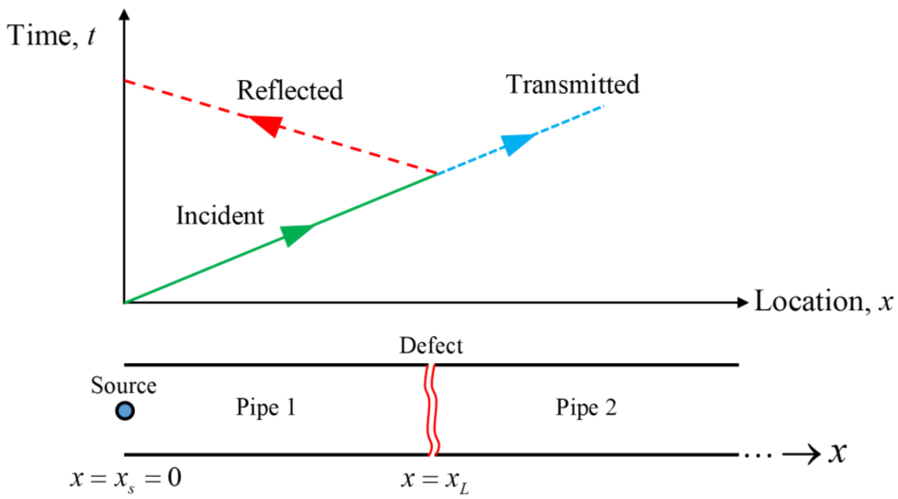

Figure 1: Configuration of the setup with a source at $x = x_s = 0$ and a discrete defect at $x = x_L$.

where R and T represent the reflection and transmission coefficients. Substituting Eqs. [9] and [10] in Eq. [3] and solving for $h(\omega, x)$ gives

For pipe 1: $$h(\omega,x) = \frac{i\mu}{i\omega A\left(\frac{g}{a_{ve}^2}\right)}\left[\exp(-i\mu x) - R\exp(i\mu x)\right] = \gamma\left[\exp(-i\mu x) - R\exp(-i\mu x)\right] \quad [11]$$

where $$\gamma = \frac{i\mu}{i\omega A\left(\frac{g}{a_{ve}^2}\right)}$$

For pipe 2: $$h(\omega,x) = \gamma T\exp(-i\mu x) \quad [12]$$

If the defect is a leak, the boundary conditions at $x = x_L$ are

$$h(x_L^-) = h(x_L^+)$$
$$\Rightarrow \exp(-i\mu x_L) - R\exp(i\mu x_L) = T\exp(-i\mu x_L) \quad [13]$$

and

$$q(x_L^-) - q_L = q(x_L^+)$$
$$\Rightarrow \exp(-i\mu x_L) + R\exp(i\mu x_L) - q_L = T\exp(-i\mu x_L), \quad [14]$$

where q_L is the discharge from the leak and the superscripts ($+$ and $-$) denotes left and right side of the leak. Addition of Eq. [13] and [14] results

$$T = 1 - \frac{q_L}{2}\exp(i\mu x_L) \quad [15]$$

and the substitution of Eq. [15] in Eq. [13] gives

$$R = \frac{q_L}{2}\exp(-i\mu x_L). \quad [16]$$

From the linearized orifice relation, it is also known that

$$q_L \cong \frac{Q_o}{2H_o}h(x_L) = \frac{Q_o}{2H_o}\gamma\left(\exp(-i\mu x_L) - R\exp(i\mu x_L)\right) \quad [17]$$

With this relation, Eq. [16] can be simplified to

$$R(\mu) = \frac{Z/4Z_L}{1+Z/4Z}\exp(-2i\mu x_L). \quad [18]$$

where $Z=\gamma$ is the impedance of the pipe wall and $Z_L = 2H_o/Q_o$ is the impedance of the leak. For a special case where there is no friction and no viscoelasticity effect, $\mu = \omega/a_e$, where a_e is the wave speed in elastic pipe (Wylie et al. 1993), and

$$R(\omega) = \frac{Z/Z_L}{4+Z/Z_L}\exp(-2i\omega x_L). \quad [19]$$

Taking the Inverse-Fourier transform (F^{-1}) of Eq. [19] gives the reflection coefficient through the leak as a function of time as

$$\begin{aligned} R(t) &= F^{-1}(R(\omega)) = \frac{1}{2\pi}\int R(\omega)\exp(i\omega t) \\ &= \frac{1}{2\pi}\frac{Z/Z_L}{4+Z/Z_L}\int \exp\left(i\omega\left(t-\frac{2x_L}{a}\right)\right)d\omega. \end{aligned} \quad [20]$$

Since,

$$\frac{1}{2\pi}\int \exp(i\omega(x-a)) = \delta(x-a) \quad [21]$$

$$\Rightarrow R(t) = \frac{Z/Z_L}{4+Z/Z_L}\delta\left(t-\frac{2x_L}{a}\right) \quad [22]$$

Thus, the pressure head response of the system at the source $x = x_s = 0$ can be determined by substituting Eq. [22] in Eq. [11] as

$$\begin{aligned} h(x_s, x_L, \omega) &= Z(1-R(w)) \\ &= Z\left[1 - s_L\exp\left(-2i\frac{\omega}{a}x_L\right)\right] \end{aligned} \quad [23]$$

in frequency domain and

$$H(x_s, x_L, t) = Z\left[\delta(t) - s_L\delta\left(t-\frac{2x_L}{a}\right)\right] \quad [24]$$

in time domain, where $s_L = (Z/Z_L)/(4+Z/Z_L)$. Eqs. [23] and [24] are the analytical solutions of the one-dimensional wave equation in time and frequency for the unbounded system wherein the reflections from the boundaries are not taken into account.

2.2 Application of time-reversal technique for leak localization

Consider $H_m(x_s,x_L,t)$ is the impulse response of the system which can be obtained experimentally or computed analytically using Eq. [24]. The time reversal operation will change the direction of this signal in time which can be expressed as $H_m(x_s,x_L,-t)$. Meaning, the values recorded at the end become the first and the first ones go to the end. If this reversed signal is remitted from the source, the response of the system can be computed by convolving the inverted signal with the response function of the system as

$$B(x_s,\hat{x}_L,t) = \underbrace{H_m(x_s,x_L,-t)}_{\text{Term I}} * \underbrace{H(x_s,\hat{x}_L,t)}_{\text{Term II}} \\ = \int_{-\infty}^{\infty} H_m(x_s,x_L,t-\tau)H(x_s,\hat{x}_L,-\tau)\,d\tau \qquad [25]$$

where $*$ represent the convolution. Notice that, Term I contains information on the true leak location. Whereas, in term II, $\hat{x}_L$ is an estimate of x_L. This signal has a well-defined maximum at $t=0$ and $x_L=\hat{x}_L$ (de Mello et al. 2016). This property, in this work, is utilized to estimate the leak location. The procedure to perform proposed the leak localization method is as follows:

i. Excite the system from one of the access/entry location by imposing an impulse and measure the impulse response function of the system.
ii. Estimate the wave speed and truncate the signal up to $t \le 2L_b/a$, where L_b is the location of the dynamic boundary.
iii. Flip the truncated signal, make a guess for Z_L to compute s_L and convolve with Eq. [24] for all possible leak locations $\hat{x}_L^k(k=1,\ldots,K)$ to estimate $B^k(x_s,\hat{x}_L^k,t=0)$, where superscript K denotes the total number of possible leak locations.
iv. Plot $B^k(x_s,\hat{x}_L^k,t=0)$ with respect to $\hat{x}_L^k$ wherein the maximum value of B^k corresponds to the true leak location.

In frequency domain, the time-reversal operator simply becomes the complex conjugate of the impulse response function and the convolution operation becomes the multiplication. This allows writing Eq. 25 in frequency domain as

$$B(x_s,\hat{x}_L,\omega) = h_m^C(x_s,x_L,\omega)\times h(x_s,\hat{x}_L,\omega) \qquad [26]$$

where C is the complex conjugate, $h_m^C(x_s,x_L,\omega)$ is the measured signal and $h(x_s,\hat{x}_L,\omega)$ is the analytical expression given by Eq. [23]. Eq. [23] shows that the output function B will only be maximum when the frequency content in both terms are in phase. Thus, again, the leak location that maximizes B (in Eq. [26]) will corresponds to the exact leak location. In the next section, the proposed method is tested numerically in both time and frequency domain for one leak case.

3 NUMERICAL CASE STUDY

Consider a frictionless elastic pipe of length $L=500\mathrm{m}$ wherein the source is located at $x=x_s=0$. The internal diameter of the pipe $D=0.5\mathrm{m}$ and the theoretical wave speed $a=1200\mathrm{m/s}$. One leak of size $0.0098\,\mathrm{m}^2$ (i.e. 5% of pipe area) is located at $x_l = 300\mathrm{m}$ from the source. An impulse of the order of Z is imposed from the source location and the response of the system is computed at the same location. The computed and time-reversed signals are shown in Figure 2. Gaussian white noise with zero mean and fixed variance is also added to the signals. The noise variance is obtained by the following expression (Keramat et al. 2017)

$$\sigma = \Delta h_l \times 10^{-\frac{SNR}{20}} \qquad [27]$$

where σ is the standard deviation of noise and Δh_l is the pressure drop caused by the leak.

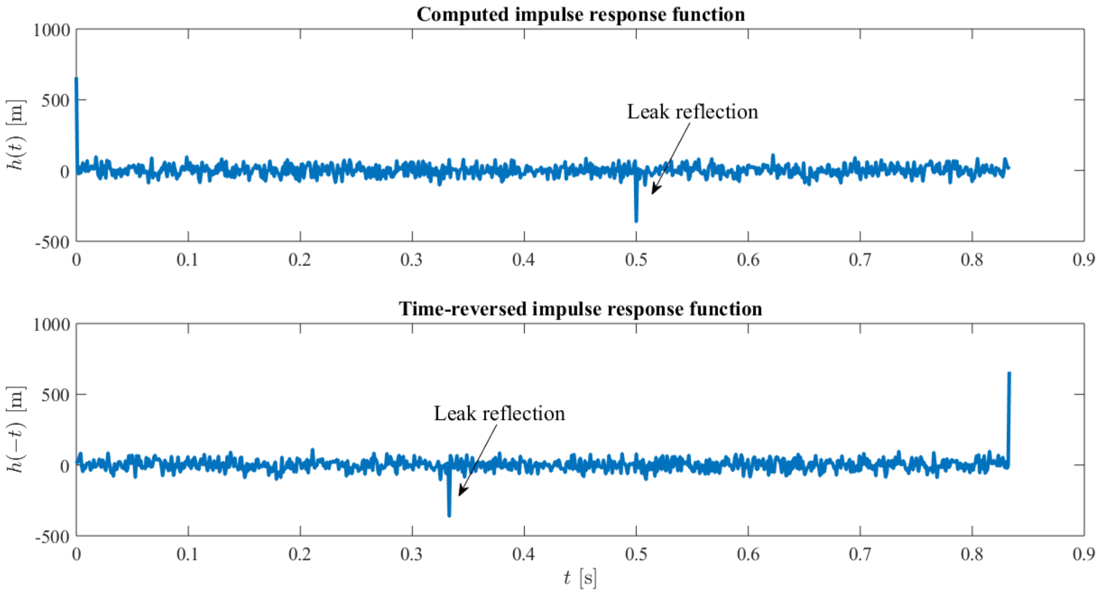

Figure 2: Measured pressure head response of the system for a given impulse with SNR = 15dB.

To localize the leak in time domain, $H_m(-t)$ is convolved with Eq. [24] for all $\hat{x}_L^k$ starting from 0 to 500m wherein the size of the leak is kept the same as of true leak size. To localize the leak in frequency domain, the complex conjugate of the original signal is multiplied with Eq. [23] for the same leak candidates. It is known that the size of the leak is also an unknown parameter and need to be estimated. However, the present work only focuses on the localization of leaks to access the workability of TR for leak localization. Future work will address the issue of leak size estimations. Figure 3 presents the plot between $B^k\left(x_s, \hat{x}_L^k, t=0\right)$ verses $\hat{x}_L^k$ for both (a) time domain and (b) frequency domain method wherein the peak represents the estimated leak location. In both domains, the leak is successfully localized. It is also interesting to note that the order of the magnitude of the output function $B^k\left(x_s, \hat{x}_L^k, t=0\right)$ is higher in frequency domain than in time domain. This information can be helpful to deploy the method in frequency domain for the case when the SNR is very low.

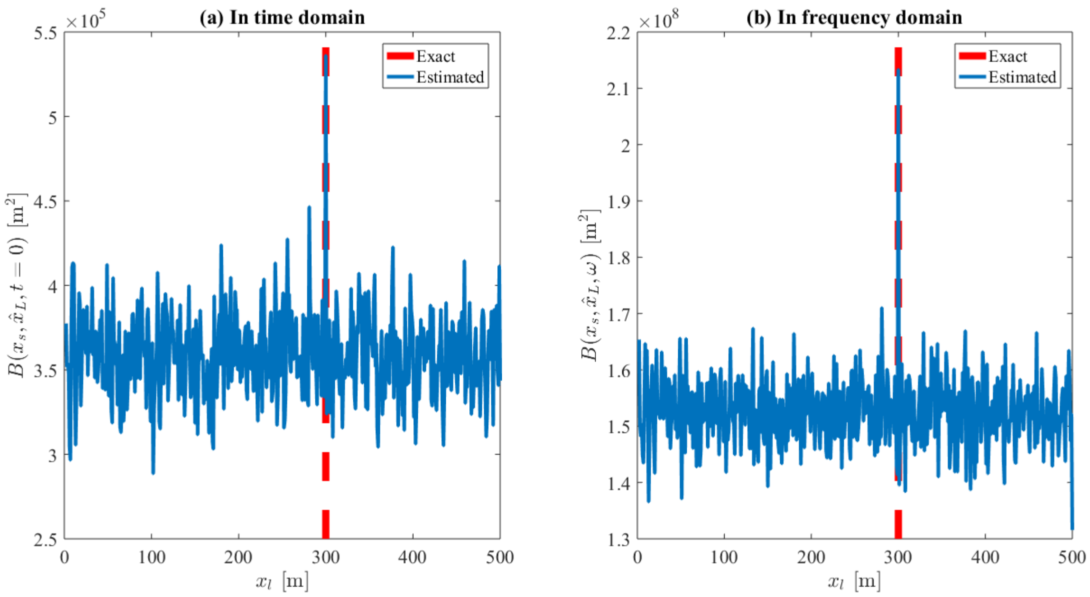

Figure 3: Estimated leak location in (a) time domain and (b) frequency domain.

4 CONCLUSIONS AND FUTURE WORK

Pipeline leak localization in the presence of measurement noise using time-reversal technique is proposed and tested numerically. An analytical solution for the one-dimensional transient wave is derived for the unbounded system. Meaning, the solution only takes into account the reflections that arrive prior to the reflection from any known dynamic boundary. The solution allows avoiding the excessive complications caused by the dynamic boundary conditions. The analytical solution is then combined with the time-reversal technique by convolving the impulse response function of the system, influenced by the measurement noise, with the analytical solution. At $t = 0$, the output of the convolution integral has a well-defined maximum only for the case when both the time-reversed and analytical signals have the same information of the leak location. This unique property is utilized to estimate the leak location. The proposed method is tested numerically in both time and frequency domain for one leak case in the presence of Gaussian white noise. The leak is successfully localized.

The proposed method has several advantages over other available methods. It is simple in terms of formulation and doesn't requires any complex computation of Hessian and Jacobian Matrices like other inverse methods (Colombo et al. 2009). It is robust in terms of leak localization in the presence of noise. Its implementation does not require the modelling of the dynamic boundaries. However, the main disadvantage of the proposed method is that it may not provide accurate results when the first reflection coefficient from the leak is very small as compare to the noise level. In such scenarios, methods which incorporate the boundary conditions are more suitable. Besides pros and cons, there are few important concerns which are not addressed in this work. First, the estimation of leak sizes should be made either together or separately from the leak localization procedure. Second, the sensitivity analysis with random modelling parameters (e.g. wave speed uncertainty) is also required to test the accuracy of the method. Third, the method need to be tested in the presence of frictional and viscoelastic damping. All of these aforementioned concerns together with experimental validation will be presented in the future work.

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